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4 INPUT-OUTPUT TABLES AND REGIONAL INCOME ACCOUNTS

Introduction

This chapter presents the input-output table as an accounting system for an economy. Labeled “hypothetical”, the numeric example is actually a 5-industry aggregation of the detailed table for Georgia in 1970 (Schaffer 1976). (Although out-of-date, it fits the style of most regional input-output tables in current use in the United States. It also fits the text, which is a slight revision of that used to explain the Georgia system.) First we look at the table as a whole; then we examine in more detail the quadrant of the table which reports the income and product accounts for this regional economy.

The regional transactions table

A regional input-output model traces the interactions of local industries with each other, with industries outside the region, and with final demand sectors. The central element in this model is a regional transactions table such as that shown in Table 4.1. This table records transactions between five broad industries, three final-payments sectors, and three final-demand sectors. (The original presentation was of transactions between 50 industries, six final-payments sectors, and 6 final-demand sectors.)

Table 4.1 Hypothetical interindustry transactions

<table>
<thead>
<tr>
<th>Selling industry</th>
<th>Buying Industry</th>
<th>Extraction (1)</th>
<th>Construction (2)</th>
<th>Manufacturing (3)</th>
<th>Trade (4)</th>
<th>Services (5)</th>
<th>Total local inputs (6)</th>
<th>Household expenditures (7)</th>
<th>Other local final demand (8)</th>
<th>Exports (9)</th>
<th>Total final demand (10)</th>
<th>Total demand (11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraction</td>
<td>183</td>
<td>31</td>
<td>599</td>
<td>6</td>
<td>73</td>
<td></td>
<td>892</td>
<td>99</td>
<td>88</td>
<td>596</td>
<td>782</td>
<td>1674</td>
</tr>
<tr>
<td>Construction</td>
<td>14</td>
<td>1</td>
<td>43</td>
<td>14</td>
<td>293</td>
<td></td>
<td>364</td>
<td>0</td>
<td>1803</td>
<td>353</td>
<td>2155</td>
<td>2520</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>142</td>
<td>414</td>
<td>1390</td>
<td>110</td>
<td>356</td>
<td></td>
<td>2412</td>
<td>1275</td>
<td>1130</td>
<td>9344</td>
<td>11750</td>
<td>14162</td>
</tr>
<tr>
<td>Trade</td>
<td>52</td>
<td>224</td>
<td>520</td>
<td>72</td>
<td>257</td>
<td></td>
<td>1126</td>
<td>2563</td>
<td>161</td>
<td>970</td>
<td>3695</td>
<td>4820</td>
</tr>
<tr>
<td>Services</td>
<td>102</td>
<td>221</td>
<td>862</td>
<td>558</td>
<td>1990</td>
<td></td>
<td>3733</td>
<td>4262</td>
<td>523</td>
<td>2828</td>
<td>7613</td>
<td>11347</td>
</tr>
<tr>
<td>Total local inputs</td>
<td>493</td>
<td>891</td>
<td>3415</td>
<td>760</td>
<td>2969</td>
<td></td>
<td>8527</td>
<td>8199</td>
<td>3705</td>
<td>14091</td>
<td>25995</td>
<td>34523</td>
</tr>
<tr>
<td>Households</td>
<td>595</td>
<td>665</td>
<td>3696</td>
<td>2385</td>
<td>4603</td>
<td>11944</td>
<td></td>
<td>100</td>
<td>2524</td>
<td>0</td>
<td>2623</td>
<td>14567</td>
</tr>
<tr>
<td>Other payments</td>
<td>261</td>
<td>191</td>
<td>1624</td>
<td>1365</td>
<td>2402</td>
<td>5842</td>
<td>(3789.2)</td>
<td>(943.2)</td>
<td>(1097.5)</td>
<td>0</td>
<td>5842</td>
<td></td>
</tr>
<tr>
<td>Imports</td>
<td>325</td>
<td>773</td>
<td>5428</td>
<td>311</td>
<td>1372</td>
<td>8209</td>
<td>3778</td>
<td>1057</td>
<td>-12994</td>
<td>-8159</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Total final payments</td>
<td>1181</td>
<td>1629</td>
<td>10747</td>
<td>4060</td>
<td>8378</td>
<td>25995</td>
<td>3878</td>
<td>3581</td>
<td>-12994</td>
<td>-5536</td>
<td>20459</td>
<td></td>
</tr>
<tr>
<td>Total inputs</td>
<td>1674</td>
<td>2520</td>
<td>14162</td>
<td>4820</td>
<td>11347</td>
<td>34523</td>
<td>12077</td>
<td>7285</td>
<td>1097</td>
<td>20459</td>
<td>54982</td>
<td></td>
</tr>
</tbody>
</table>
Each row in this table accounts for the sales by the industry named at its left to the industries identified across the top of the table and to the final consumers listed in the right-hand section of the table. Intermediate goods are sold to local industries for use in producing other products while finished goods are sold to final consumers. Goods exported from the region to other parts of the nation and the world are listed under exports in the final-demand section, regardless of their stage of production. The sum of a row is the total output or total sales of an industry.

Thus, sales by the extraction industry (a combination of agricultural, forestry, fishing, and mining industries) are shown in row one of Table 4.1. Of the total output worth $1,674 million, over 35 percent is sold to light manufacturing (which processes it for further sale), and over 35 percent is sold outside the region. The remaining sales are largely to other industries within the broad extractive industry itself.

Each column in Table 4.1 records the purchases, or inputs, of the industry identified at the top of the column from the industries named at the left. Payments by the industry to employees, holders of capital, and governments are contained in the first two rows of the final-payments section of the table. These payments constitute the "value added" by the industry in question. Purchases from industries outside the region are identified in the last row of the final-payments section and are called "imports." These imports may be either of goods not produced at all in the region or of goods produced in quantities insufficient to meet local needs. The sum of the entries in each column represents the total purchases by the industry in question. Since profits, losses, depreciation, taxes, etc., are recorded in the table as final payments, the total purchases and payments must equal total sales. Inputs equal outputs; hence the term "input-output."

For example, the purchases and payments of the extractive industry are shown in column one of Table 4.1. Since this industry is almost 90 percent agriculture, the column reflects large intrasector transactions (purchases of feeder stocks, baby chicks, grains, etc.), substantial purchases from light manufacturing (feeds), and a large payment to households for labor and proprietors' income. Local farmers also import from outside the state large amounts of feeds and other supplies. Notice that the total input is the same as the total demand identified in row one.
Now, with this brief introduction to a regional transactions table, let us look at the table as an accounting system for an economy. Figure 4.1 shows an input-output table in skeleton form and divided into four quadrants. Quadrant I describes consumer behavior, identifying consumption patterns of households and such other local final users of goods as private investors and governments. Another important part of Quadrant I is the export column, which shows sales to other industries and consumers outside the regional economy. Since these goods would not normally reappear in the region in the same form, these sales are regarded as final. According to economic-base theory, in which final demand is the motivating force in an economy, we would look in this quadrant for activity-generating forces and we would especially examine the government and export sectors.

Quadrant II depicts production relationships in the economy, showing the ways that raw materials and intermediate goods are combined to produce outputs for sale to other industries and to ultimate consumers. This is the most important quadrant in an input-output table. For regions, it typically ranges in size between 30 and 500 industries. Quadrant II is the basis for the input-output model itself.

Quadrant III shows incomes of primary units of the economy, including
the incomes of households, the depreciation and retained earnings of industries, and the taxes paid to various levels of government. These payments are also called value added; since they are so hard to identify individually, these incomes are frequently recorded as one value-added row. The quadrant also includes payments to industries outside the economy for materials and intermediate goods which are imported into the region. Since all of these payments to resource owners and to outsiders leave the industrial system of the region, they are called "final payments."

Quadrant IV identifies primarily nonmarket transfers between sectors of the economy and might properly be labeled the "social transfers" quadrant. Here we see gifts, savings, and taxes of households; we see the surpluses and deficits of governments and their payments to households and intergovernmental transfers. The quadrant also typically includes purchases by final-demand sectors from industries outside the region.

Now, to make a major point about the hazards of aggregation, let's look at the table as originally presented, as a picture of the Georgia economy in 1970. Out of a total output of over $34 billion, Georgia's manufacturing output in 1970 was valued at over $14 billion and its service output at over $11 billion, indicating that Georgia's economy was dominated by the manufacturing and service industries. Even so, Georgia was not a major manufacturing or service economy by national standards, as can be seen in the following comparison of the industrial origins of value added in Georgia and the United States (Schaffer 1976)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Percent of value added</th>
<th>Georgia</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, mining</td>
<td>4.2</td>
<td>5.2</td>
<td></td>
</tr>
<tr>
<td>Construction</td>
<td>4.4</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>Manufacturing</td>
<td>26.0</td>
<td>28.9</td>
<td></td>
</tr>
<tr>
<td>Transportation, utilities</td>
<td>7.7</td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>Trade</td>
<td>18.3</td>
<td>14.6</td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>25.6</td>
<td>26.9</td>
<td></td>
</tr>
<tr>
<td>Government</td>
<td>13.7</td>
<td>10.7</td>
<td></td>
</tr>
</tbody>
</table>

Georgia had larger contributions to value added from trade and government than did the nation, and smaller contributions from the extractive industries, construction, manufacturing, utilities, and services. This deviation from the national pattern is an expression of the region's modest stage of development and its central position in the Southeast.

But this observation also shows that the "importance" of an industry is completely dependent on the definitions and aggregation patterns employed in constructing a table. By enlarging the table and altering sector definitions, we could change the apparent importance of industries. For example, by combining the agricultural industries with the food-processing industry (normally in manufacturing) we could make the "agriculture-based" industry larger than any of the components of the "trade" or "service" industries. In fact, in the 29-industry version (not shown here) of the Georgia table, the five largest industries in terms of output are: 1) trade, 2) finance, insurance and real estate, 3) services, 4) textile mill products, and 5) transportation equipment.

A second interesting item in Table 4.1 is the gross product of Georgia. Analogous in concept to the gross national product, gross regional product (GRP) can be defined as total production without duplication, or as the economic product of all factors of production...
residing in the region. It can also be seen as the total final payments (adjusted for imports) in the region, 20.459 billion dollars. Alternatively, it is also the total final demand by ultimate consumers of the region's products (net of imports).

In summary, an input-output table traces the paths by which incomes flow through the economy. Quadrant I is where the spending cycle begins and is where finished goods go to satisfy the needs of final consumers. Quadrant III is where the production cycle starts, with households and other resource owners, including governments, receiving payments for their contributions to the production process. Quadrant II traces production relationships, describing the technology of production in the economy. It outlines the market sector of the economy. Quadrant IV identifies nonmarket flows of money, showing purchases of labor inputs by governments, taxes paid by households, surpluses and deficits of governments, and transfers between governments and other governments and people.

**Income and product accounts**

The input-output table embodies not only measures of gross regional product but also a summary set of social, or income and product, accounts for the region. Like the input-output table itself, these accounts are part of a double-entry accounting system for the economy. In the same way that a businessman uses his accounts to develop a consolidated income statement for his firm, the economist uses income and product accounts to measure the performance of the economy and to compare the behavior of parts of the economy against other standards.

Table 4.2 is the transactions table rearranged to emphasize Quadrant IV, the sector in which social accounts are traced. This social-accounts table completely ignores the flows of intermediate products through the production quadrant and suppresses the details of the other quadrants. It emphasizes (1) the total final payments to resource owners for their contributions to production, (2) the aggregate demand for final products, and (3) the transfers which take place between primary units of the economy.

We have slightly rearranged the table. The row showing purchases from nonlocal industries (imports) has been moved above the final-payments rows. A row for transfers to households has been added to account for nonproductive money transfers to persons. And the one row for other payments in Table 4.1 has been expanded into four to show the details of final payments and transfers.

Six accounts are outlined in the table. The receipts side of the household account is shown in the household-income and household-transfers rows, which total to be personal income; the payments side is detailed in the household-expenditures column. The saving and investment account is shown in the capital-residual row (retained earnings, depreciation, savings) and the investment column. Local, state, and federal government accounts are shown in their rows and columns. And the rest-of-the-world account is shown in the row labeled "purchases from nonlocal industry" and the column "net exports." By placing these accounts into one matrix, we gain both economy in presentation and a feeling for their commonality.
Gross state product (GSP) may be measured in two ways, the incomes approach and the expenditures approach. Let us start with the expenditures approach.

Using expenditures, we define GSP as state output at market value as measured through the expenditures of final consumers. This approach accounts for the final demand for Georgia's product by four groups of consumers: households, investors, governments, and private units outside the state economy. In Table 4.2, GSP is seen as total purchases of goods and services for final consumption, $20,459 million. In 1970, this was 2.1 percent of GNP. In comparison to expenditures for GNP, Georgia spent less of her gross product on personal consumption (59.0 percent in contrast to 62.9 percent for the nation), less on private investment (10.7, 13.5), and less on local and state government (11.3, 12.2); she made up for this in terms of federal defense expenditures (8.4, 7.5), other federal expenditures (5.0, 3.6), and net private exports (5.3, 0.4).

Table 4.2 Income and product accounts for Georgia, 1970

<table>
<thead>
<tr>
<th>Account receiving/making payment</th>
<th>Sales to processing sectors</th>
<th>Household expenditures</th>
<th>Private investment</th>
<th>Expenditures of governments</th>
<th>Net exports</th>
<th>Total final demand</th>
<th>Total receipts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases from local processors</td>
<td>8,527.2</td>
<td>8,199.3</td>
<td>1,400.4</td>
<td>460.9</td>
<td>432.3</td>
<td>1,057.0</td>
<td>353.9</td>
</tr>
<tr>
<td>Purchases from nonlocal industry</td>
<td>8,159.3</td>
<td>3,777.8</td>
<td>802.1</td>
<td>138.8</td>
<td>115.8</td>
<td>-12,993.8</td>
<td>-8,159.3</td>
</tr>
<tr>
<td>Total purchases from industry</td>
<td>16,686.5</td>
<td>11,977.1</td>
<td>2,202.5</td>
<td>599.7</td>
<td>548.1</td>
<td>1,057.0</td>
<td>353.9</td>
</tr>
<tr>
<td>Household income</td>
<td>11,882.6</td>
<td>99.7</td>
<td>790.7</td>
<td>372.7</td>
<td>671.0</td>
<td>689.3</td>
<td>2,623.4</td>
</tr>
<tr>
<td>Total purchases of goods and services</td>
<td>28,569.1</td>
<td>12,076.8</td>
<td>2,202.5</td>
<td>1,390.4</td>
<td>920.8</td>
<td>1,728.0</td>
<td>1,043.2</td>
</tr>
<tr>
<td>Household transfers</td>
<td>61.0</td>
<td>51.3</td>
<td>190.7</td>
<td>209.0</td>
<td>848.0</td>
<td>1,299.0</td>
<td>1,360.0</td>
</tr>
<tr>
<td>Capital residual</td>
<td>3,019.1</td>
<td>871.6</td>
<td></td>
<td></td>
<td></td>
<td>-1,688.2</td>
<td>-816.6</td>
</tr>
<tr>
<td>Local government income</td>
<td>480.3</td>
<td>377.9</td>
<td></td>
<td></td>
<td></td>
<td>445.9</td>
<td>47.3</td>
</tr>
<tr>
<td>State government income</td>
<td>858.8</td>
<td>341.9</td>
<td></td>
<td></td>
<td></td>
<td>22.1</td>
<td>408.8</td>
</tr>
<tr>
<td>Federal government income</td>
<td>1,533.9</td>
<td>2,197.8</td>
<td></td>
<td></td>
<td></td>
<td>19.1</td>
<td>533.5</td>
</tr>
<tr>
<td>External transfers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2,750.4</td>
<td>4,284.3</td>
</tr>
<tr>
<td>Total outlay</td>
<td>34,522.2</td>
<td>15,866.0</td>
<td>2,202.5</td>
<td>1,463.8</td>
<td>1,576.5</td>
<td>1,937.0</td>
<td>2,347.3</td>
</tr>
</tbody>
</table>

Using incomes, we can arrive at a similar GSP by adding the "income receipts" of the various accounts. The major receipt is earned household or personal income, which consists of wages and salaries, other labor income, proprietors' income, and property incomes. Including business transfer payments (primarily bad debts) and social security contributions, this amounts to $14,567 million, or 71.2 percent of GSP; the corresponding
national figure is 75.2 percent. The "capital residual," or gross business saving, of processors is $3,019 million and comprises 14.8 percent of GSP, which corresponds to 9.4 percent in the nation. The capital-residual row of Table 4.2 includes two transfers worth noting: one is personal savings; the other is a negative entry of $1,688 million in the exports column. This "export" accounts for the surpluses and deficits of the various governments and the outside world. Much of it represents flows of retained earnings and capital consumption allowances to the nonresident owners of branch plants in Georgia.

The third receipt to be added to GSP is local government income from the processing sector. At $480 million, this figure accounted for 2.3 percent of GSP. The next largest income of local governments in Georgia was a set of intergovernmental transfers from the state government (much of which is offset by a similar transfer from the federal government to the state). The deficits of local governments are shown as an "export" (primarily bonds) worth $112.4 million.

The fourth receipt to be counted as part of GSP is state government income from the processing sector of $859 million. Combined state and local revenues from industrial sources are 6.5 percent of GSP, compared to 8.5 percent on the national level. Notice that the federal government still spent $534 million more in Georgia than it received in taxes, accounted for largely through defense expenditures.

In sum, total receipts and payments by each of the six final sectors in the economy were $25,393 million. This figure is $4,934 million in excess of GSP. Where quadrant II shows intermediate transactions in the processing sector, the transfers quadrant records duplicative transactions in the social or political sector.

Summary

A state input-output table accounts for flows of monies through the state, showing details regarding consumer behavior, the technology of production, incomes, and social transfers. The transfers quadrant of a table can be slightly modified to show the details presented in the more traditional income and product accounts.

The social accounts are useful in two ways. One is in comparisons between economies; a brief contrast of the Georgia and U.S. economies has been sketched here and Georgia has been found to be strong in trade and government and slightly below the national pattern in manufacturing and services. The other way is in comparisons over time. But to show performance over time, social accounts and input-output tables must be constructed on a regular basis by state agencies.

Appendix 1: DVD data sources: the Regional Economic Information System

One of the best sources of regional data is the Regional Economic...
Information System (REIS), produced by the Regional Economic Measurement Division of the Bureau of Economic Analysis.

Many Web sites have information from REIS available for downloading, but the most common source for the heavy user is the DVD disk. Published annually in May, it contains data for states, regions, and counties from 1969 forward, with a two-year lag. (Thus, the May 2009 disk contains data from 1969 to 2007.) Now, the data contained in the disk and much more are also available free of charge from http://www.bea.gov/regional/docs/reis2007dvd.cfm. (Note that this web page is hard to find when starting from bea.gov – this may signal the demise of this product due to budget constraints.)

For counties, the disk provides data on personal income and its sources, employment (in broad industry categories), an economic profile, transfer payments, and agricultural output. In addition it contains data on the journey to work between counties in 1970, 1980, 1990, and 2000, as well as annual estimates of gross commuter incomes.

More detailed data on journey to work and place of work as developed in the 2000 Census can be found at http://www.census.gov/population/www/socdemo/journey.html.

Appendix 2: Measures of regional welfare: Personal Income

The problem with GSP estimates

Gross state product accounts have been established for a number of states and using a variety of methods. In Hawaii, with a long history of independence, accounts have been constructed along exactly the same lines as a nation (in fact, some of the people involved in these accounts were close associates of members of the accounts team in the U.S. Bureau of the Census.

In most other states, however, the method has been a short-cut method known as the Kendricks-Jaycox method and involves estimates based on ratios.

Estimates of Gross State Product have been assembled from all sources available to the Regional Economic Measurement Division. These are available for selected years on the Regional Economic Information System. One nice thing about this central source is that the accounts for the 50 states are consistent with the accounts for the nation as a whole. For details and availability, see http://www.bea.gov/regional/index.htm#gsp.

The following statement shows the formal difference between "Gross Regional Product" (GRP) and "Gross Domestic Regional Product," (GDRP) which parallels the difference between GNP and GDP. This equation shows development of GRP as GDRP adjusted for net factor payments (including profits).
GROSS DOMESTIC REGIONAL PRODUCT

LESS INTEREST AND DIVIDENDS TO NONRESIDENTS

PLUS INTEREST AND DIVIDENDS FROM REST OF WORLD

LESS WAGES Earned by IN-COMMUTERS

PLUS WAGES Earned by OUT-COMMUTERS

EQUALS GROSS REGIONAL PRODUCT

The widespread use of personal income estimates

The problems associated with estimating flows of capital income from one area to another are severe. We just can't develop accurate statistics on ownership of large multi-state and multi-national corporations and thus on the flows of dividends and interest across state boundaries.

As a result of this handicap, we normally consider personal income as a better measure of individual welfare. Considerable data on journey to work and commuting across county boundaries over the last three decennial censuses has permitted the Regional Economic Measurement Division to estimate adjustments for residency, which solves part of the GRP/GDRP problem.

GROSS DOMESTIC REGIONAL PRODUCT is the total value of goods and services produced by factors of production owned by residents of the region; GROSS DOMESTIC REGIONAL PRODUCT is the total value of production in the region.

In the United States, GNP is greater than GDP -- we own more abroad than others do here. In Hawaii, GRP is less that GDRP -- natives own less abroad than others own in Hawaii. In other states, the question has not been answered.

The point of this is that, since people always confuse lengthy titles, we just go ahead and confuse things to begin with and rename gross domestic state product as gross state product!

GSP = GDSP
5 THE LOGIC OF INPUT-OUTPUT MODELS

Introduction

The logic of input-output models is analogous to that of economic-base models, and requires only an extension of mathematical sophistication from simple algebra to matrix algebra. A piece of cake. The complexities of input-output models come in devising schemes to manage the massive data sets required to build them.

We proceed here to repeat the process evolved in Lecture 3 to develop the simplest of models, a plain, square, industry-by-industry model. In the next chapter, we will develop the multiplier system associated with this model. After this work on the model and its uses, we will return to the process of building an input-output system. By then we will have manipulated the system sufficiently to easily understand the more complex commodity-by-industry accounting system now in current use by the United States and recommended by the United Nations.1

The rationale for a model: analysis vs. description

While the transactions table describes the economy and yields interesting bits of information for a particular point in time, in itself it has no analytic content. That is, it does not permit us to answer questions concerning the reaction of the economy to change. Let the transactions table represent the economy in equilibrium and subject it to a shock, say an increase in tourism or a cutback in defense expenditures. When the repercussions of the shock have moved through the economy, what will be its new "equilibrium position?" In other words, which industries will be larger or smaller and whose income or employment will have changed? Such analysis requires an economic model, which we now proceed to construct.

Preparing the transactions table: closing with respect to households

As we shall see, it is important to include in the interindustry structure (Quadrant II) all economic activities which make buying decisions primarily on the basis of their incomes. These activities are called endogenous since their behavior is determined within the system. Other activities, such as federal government expenditures or exports, are based on decisions made outside the system and so are called exogenous activities. Activities which are labeled "industries" are normally considered endogenous and those which are labeled "final-demand sectors" are normally considered to be exogenous. But sometimes it is not so easy to classify activities.

The household sector is a case in point. While traditionally classified as a final-demand sector, it is frequently treated in regional economic models as an "industry." Households sell labor, managerial skills, and privately owned resources; they receive in return wages and salaries, dividends, rents, proprietors' income, etc. And to produce

---

1 My favorite source for this logic is the dominant book on input-output models of the 1960's (Chenery and Clark 1959). It contains almost all we needed until the advent of commodity-by-industry accounts. I recommend it.
these resources, they buy food, clothing, automobiles, housing, services, and other consumer goods. Exceeded in total expenditures only by the manufacturing sector, the household sector is obviously a critical part of the Georgia economy or of any area economy, for that matter. So we move the household row and column into the interindustry part of the transactions table and treat households as another industry. The household sector becomes the sixth "industry" in the aggregated table repeated as revised in Table 5.1.

### Table 5.1 Hypothetical interindustry transactions with endogenous household sector

<table>
<thead>
<tr>
<th>Selling sector</th>
<th>Buying sector</th>
<th>Extraction (1)</th>
<th>Construction (2)</th>
<th>Manufacturing (3)</th>
<th>Trade (4)</th>
<th>Services (5)</th>
<th>Household expenditures (7)</th>
<th>Total industry demand (6)</th>
<th>Other local final demand (8)</th>
<th>Exports (9)</th>
<th>Total final demand (10)</th>
<th>Total demand (11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraction</td>
<td>(1)</td>
<td>183</td>
<td>31</td>
<td>599</td>
<td>99</td>
<td>991</td>
<td>88</td>
<td>596</td>
<td>684</td>
<td>1674</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Construction</td>
<td>(2)</td>
<td>14</td>
<td>1</td>
<td>43</td>
<td>14</td>
<td>293</td>
<td>0</td>
<td>364</td>
<td>1803</td>
<td>353</td>
<td>2155</td>
<td>2520</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>(3)</td>
<td>142</td>
<td>414</td>
<td>1390</td>
<td>110</td>
<td>356</td>
<td>1275</td>
<td>3687</td>
<td>1130</td>
<td>9344</td>
<td>10474</td>
<td>14162</td>
</tr>
<tr>
<td>Trade</td>
<td>(4)</td>
<td>52</td>
<td>224</td>
<td>520</td>
<td>72</td>
<td>257</td>
<td>2563</td>
<td>3689</td>
<td>161</td>
<td>970</td>
<td>1131</td>
<td>4820</td>
</tr>
<tr>
<td>Services</td>
<td>(5)</td>
<td>102</td>
<td>211</td>
<td>862</td>
<td>558</td>
<td>1990</td>
<td>4262</td>
<td>7996</td>
<td>523</td>
<td>2828</td>
<td>3351</td>
<td>11347</td>
</tr>
<tr>
<td>Households</td>
<td>(7)</td>
<td>595</td>
<td>665</td>
<td>3696</td>
<td>2385</td>
<td>4603</td>
<td>100</td>
<td>12043</td>
<td>2524</td>
<td>0</td>
<td>2524</td>
<td>14567</td>
</tr>
<tr>
<td>Total local inputs</td>
<td>(6)</td>
<td>1088</td>
<td>1556</td>
<td>7110</td>
<td>3145</td>
<td>7572</td>
<td>8299</td>
<td>28770</td>
<td>6228</td>
<td>14091</td>
<td>20319</td>
<td>49090</td>
</tr>
<tr>
<td>Other payments</td>
<td>(8)</td>
<td>261</td>
<td>191</td>
<td>1624</td>
<td>1365</td>
<td>2402</td>
<td>3789</td>
<td>9632</td>
<td>(943.2)</td>
<td>1097.5</td>
<td>0</td>
<td>5842</td>
</tr>
<tr>
<td>Imports</td>
<td>(9)</td>
<td>325</td>
<td>773</td>
<td>5428</td>
<td>311</td>
<td>1372</td>
<td>3776</td>
<td>11987</td>
<td>1057</td>
<td>-12994</td>
<td>-11937</td>
<td>50</td>
</tr>
<tr>
<td>Total final payments</td>
<td>(10)</td>
<td>586</td>
<td>964</td>
<td>7051</td>
<td>1675</td>
<td>3775</td>
<td>7567</td>
<td>21619</td>
<td>3581</td>
<td>-12994</td>
<td>-9413</td>
<td>20459</td>
</tr>
<tr>
<td>Total inputs</td>
<td>(11)</td>
<td>1674</td>
<td>2520</td>
<td>14162</td>
<td>4820</td>
<td>11347</td>
<td>15866</td>
<td>50389</td>
<td>7285</td>
<td>1097</td>
<td>20459</td>
<td>54982</td>
</tr>
</tbody>
</table>

The state and local government sectors (included in "other final demand" and "other final payments" in our aggregated table) also are difficult to classify. While we leave them in the exogenous part of the table now, primarily for simplicity, they are occasionally included in the endogenous part of the table in detailed forecasting models.

### The economic model

As is now familiar, an equilibrium model is based on three sets of relations: (1) definitions or identities, (2) technical or behavioral conditions, and (3) equilibrium conditions. A model thus uses a set of assumptions to extend a description of an economy so that it can be used to trace the effects of disequilibrating forces. In the case at hand, each set of relations can be easily identified.

**Identities: the transactions table**

The state transactions table as extended above provides our set of identities: it defines the economy for the base year. Now let's express these relations in simple algebra. Let $z_{ij}$ be the sales of industry $i$ to industry $j$, $e_i$ the sales of industry $i$ to other final demand (ultimate consumers), and $q_i$ the total sales of industry $i$, or total demand for the output of the industry. Then we can
define the sales of Georgia industries in terms of the following equations:

\[
\begin{align*}
    z_{11} + z_{12} + z_{13} + z_{14} + z_{15} + z_{16} + e_1 & \equiv q_1 \\
    z_{21} + z_{22} + z_{23} + z_{24} + z_{25} + z_{26} + e_2 & \equiv q_2 \\
    z_{31} + z_{32} + z_{33} + z_{34} + z_{35} + z_{36} + e_3 & \equiv q_3 \\
    z_{41} + z_{42} + z_{43} + z_{44} + z_{45} + z_{46} + e_4 & \equiv q_4 \\
    z_{51} + z_{52} + z_{53} + z_{54} + z_{55} + z_{56} + e_5 & \equiv q_5 \\
    z_{61} + z_{62} + z_{63} + z_{64} + z_{65} + z_{66} + e_6 & \equiv q_6
\end{align*}
\]

This set of identities can be seen symbolically in the top six rows of Figure 5.1 and numerically in the five industry rows and in the household row (now “industry” six) of the transactions table (Table 5.1). Since we are now primarily concerned with Quadrant II, we have reduced Quadrant I to one column in these tables and we have dropped the various intermediate totals.

**Figure 5.1 Algebraic transactions table**

<table>
<thead>
<tr>
<th>Buying sector</th>
<th>Extraction (1)</th>
<th>Construction (2)</th>
<th>Manufacturing (3)</th>
<th>Trade (4)</th>
<th>Services (5)</th>
<th>Households (6)</th>
<th>Final demand and exports (7)</th>
<th>Total demand (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraction</td>
<td>( z_{11} )</td>
<td>( z_{12} )</td>
<td>( z_{13} )</td>
<td>( z_{14} )</td>
<td>( z_{15} )</td>
<td>( z_{16} )</td>
<td>( e_1 )</td>
<td>( q_1 )</td>
</tr>
<tr>
<td>Construction</td>
<td></td>
<td>( z_{21} )</td>
<td>( z_{22} )</td>
<td>( z_{23} )</td>
<td>( z_{24} )</td>
<td>( z_{25} ) + ( e_2 )</td>
<td>( q_2 )</td>
<td></td>
</tr>
<tr>
<td>Manufacturing</td>
<td>( z_{31} )</td>
<td>( z_{32} )</td>
<td>( z_{33} )</td>
<td>( z_{34} )</td>
<td>( z_{35} )</td>
<td>( z_{36} )</td>
<td>( e_3 )</td>
<td>( q_3 )</td>
</tr>
<tr>
<td>Trade</td>
<td>( z_{41} )</td>
<td>( z_{42} )</td>
<td>( z_{43} )</td>
<td>( z_{44} )</td>
<td>( z_{45} )</td>
<td>( z_{46} )</td>
<td>( e_4 )</td>
<td>( q_4 )</td>
</tr>
<tr>
<td>Services</td>
<td>( z_{51} )</td>
<td>( z_{52} )</td>
<td>( z_{53} )</td>
<td>( z_{54} )</td>
<td>( z_{55} )</td>
<td>( z_{56} )</td>
<td>( e_5 )</td>
<td>( q_5 )</td>
</tr>
<tr>
<td>Households</td>
<td>( z_{61} )</td>
<td>( z_{62} )</td>
<td>( z_{63} )</td>
<td>( z_{64} )</td>
<td>( z_{65} )</td>
<td>( z_{66} )</td>
<td>( e_6 )</td>
<td>( q_6 )</td>
</tr>
<tr>
<td>Final payments</td>
<td>( v_1 )</td>
<td>( v_2 )</td>
<td>( v_3 )</td>
<td>( v_4 )</td>
<td>( v_5 )</td>
<td>( v_6 )</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Imports</td>
<td>( m_1 )</td>
<td>( m_2 )</td>
<td>( m_3 )</td>
<td>( m_4 )</td>
<td>( m_5 )</td>
<td>( m_6 )</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Total inputs</td>
<td>( g_1 )</td>
<td>( g_2 )</td>
<td>( g_3 )</td>
<td>( g_4 )</td>
<td>( g_5 )</td>
<td>( g_6 )</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

In a more concise formulation, the equations may be summarized as:

\[
q_i \equiv \Sigma_j z_{ij} + e_i \tag{5-1}
\]

where the operator \( \Sigma_j \) sums sales by industry \( i \) over all industries \( j \).

As can be seen,

Figure 5.1 and the above set of equations differ in only two ways: (1) the arithmetic operators are implicit in the table; and (2) the table includes values for other final payments (\( v_j \)) and imports (\( m_j \)), completing the accounting framework.

While the above set of equations (identities) is our basic concern, a second set may reveal insight into our equilibrium problem. For the economy to be in a state of equilibrium, row and column totals must agree. (A simple fact of double-entry accounting.) We can now define total output, or supply, for industry \( j \) as \( g_j \), the sum of all intermediate purchases, local payments to factors of production, and imports:

\[
g_j \equiv \Sigma_i z_{ij} + v_j + m_j \tag{5-2}
\]

where \( \Sigma_i \) sums purchases by industry \( i \) over all industries \( j \).
Technical conditions: the direct-requirements table

Now, recall that the final-demand vector (e) is exogenous. The e’s are free to change, outside of our control. We wish to know the effects of such change on the economy as expressed by changes in output. It is obvious that little additional information can be gleaned from the transactions table. We have six equations and 48 variables, of which only six (the e’s) now have assigned values. The minimum requirement for a solution to this system is that the number of equations equals the number of unknowns; therefore, we must reduce the number of unknown variables by 42.

To do this, we introduce a set of technical conditions. Assume that the pattern of purchases identified in the base year is stable. We can now define a set of values called "direct requirements," or "production coefficients:"

\[ a_{ij} = \frac{z_{ij}}{g_j} \] (5-3)

Table 5.2 records these \( a_{ij} \) coefficients for the hypothetical model. We have simply divided each value in a column by the total inputs (output) of the industry represented in the column. These numbers show the proportions in which the establishments in each industry combine the goods and services which they purchase to produce their own products.

Table 5.2 Hypothetical direct-requirements table

<table>
<thead>
<tr>
<th>Industry</th>
<th>Extraction (1)</th>
<th>Construction (2)</th>
<th>Manufacturing (3)</th>
<th>Trade (4)</th>
<th>Services (5)</th>
<th>Household expenditures (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraction</td>
<td>10.9</td>
<td>1.2</td>
<td>4.2</td>
<td>0.1</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Construction</td>
<td>0.8</td>
<td>0.0</td>
<td>0.3</td>
<td>0.3</td>
<td>2.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>8.5</td>
<td>16.4</td>
<td>9.8</td>
<td>2.3</td>
<td>3.1</td>
<td>8.0</td>
</tr>
<tr>
<td>Trade</td>
<td>3.1</td>
<td>8.9</td>
<td>3.7</td>
<td>1.5</td>
<td>2.3</td>
<td>16.2</td>
</tr>
<tr>
<td>Services</td>
<td>6.1</td>
<td>8.8</td>
<td>6.1</td>
<td>11.6</td>
<td>17.5</td>
<td>26.9</td>
</tr>
<tr>
<td>Households</td>
<td>35.5</td>
<td>26.4</td>
<td>26.1</td>
<td>49.5</td>
<td>40.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Total local purchases</td>
<td>65.0</td>
<td>61.7</td>
<td>50.2</td>
<td>65.2</td>
<td>66.7</td>
<td>52.3</td>
</tr>
<tr>
<td>Other payments</td>
<td>15.6</td>
<td>7.6</td>
<td>11.5</td>
<td>28.3</td>
<td>21.2</td>
<td>23.9</td>
</tr>
<tr>
<td>Imports</td>
<td>19.4</td>
<td>30.7</td>
<td>38.3</td>
<td>6.4</td>
<td>12.1</td>
<td>23.8</td>
</tr>
<tr>
<td>Total final payments</td>
<td>35.0</td>
<td>38.3</td>
<td>49.8</td>
<td>34.8</td>
<td>33.3</td>
<td>47.7</td>
</tr>
<tr>
<td>Total inputs</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Notice that we can define \( z_{ij} \), the sales by industry \( i \) to industry \( j \), in another way. It can be written as \( z_{ij} = a_{ij} * g_j \). That is, if the manufacturing sector purchases 6.1 percent of its inputs from the service sector \( (a_{53}) \), and if manufacturing purchases a total of $14,162 million worth of inputs \( (g_3) \), then its purchases from the service sector would have to amount to $862 million, or 6.1 percent of $14,162 million. If the proportions in which industries buy their...
inputs remain reasonably stable over time, then we can define purchases by industry \( i \) from industry \( j \) in the future as \( z_{ij} = a_{ij} g_j \). As we shall see, this simple assumption solves our problem.

A second assumption is hidden within the definition of \( z_{ij} \) in that it is already defined as purchases from local industry \( i \) by local industry \( j \). This is best seen by looking at the derivation of the regional production coefficients, \( a_{ij} \). Each regional production coefficient is the product of a "technical production coefficient," \( p_{ij} \), which shows the proportion of inputs purchased from industry \( i \) by industry \( j \) without regard to the location of industry \( i \), and a "regional trade coefficient," \( r_{ij} \), which shows the proportion of that purchase made in the region. Symbolically, the regional production coefficient is

\[
a_{ij} = p_{ij} r_{ij}
\]

\[
a_{ij} = (z_{ij}/g_j) (z_{ij} - m_{ij}) / z_{ij}
\]

(5-4)

Here, \( z_{ij} \) is purchases from industry \( i \) by industry \( j \) without regard to the location of industry \( i \), and \( m_{ij} \) is imports of the products of industry \( i \) by industry \( j \). This point is expanded upon both in the summary included in Illustration 5.1 and in the section below on economic change.

**Equilibrium condition: supply equals demand**

Note that along the way we have implicitly stated the equilibrium condition. This condition is that, for all industries, anticipated demand equals supply, or that the sales of an industry equal its gross output:

\[
g_j = q_j
\]

(5-5)

Over any long period of time in a market economy, it is irrational to produce more than is used and impractical to consume more than is produced. Under normal conditions, an economy faced with a change in demand will react by changing supply. When anticipations are fulfilled, the economy is in a state of equilibrium.

**Solution to the system: the total-requirements table**

We can now rewrite the equation system to meet the obvious conditions for its solution. Two substitutions are required: (1) since we assume the production coefficients are stable, \( a_{ij} g_j \) can be substituted for \( z_{ij} \), and (2) since inputs must come back into balance with outputs, \( q_j \) is substituted for \( g_j \). Thus,

\[
a_{11}q_1 + a_{12}q_2 + a_{13}q_3 + a_{14}q_4 + \cdots + a_{16}q_6 + e_1 = q_1
\]

\[
a_{21}q_1 + a_{22}q_2 + a_{23}q_3 + a_{24}q_4 + \cdots + a_{26}q_6 + e_2 = q_2
\]

\[
a_{31}q_1 + a_{32}q_2 + a_{33}q_3 + a_{34}q_4 + \cdots + a_{36}q_6 + e_3 = q_3
\]

\[
a_{41}q_1 + a_{42}q_2 + a_{43}q_3 + a_{44}q_4 + \cdots + a_{46}q_6 + e_4 = q_4
\]

\[
a_{51}q_1 + a_{52}q_2 + a_{53}q_3 + a_{54}q_4 + \cdots + a_{56}q_6 + e_5 = q_5
\]

\[
a_{61}q_1 + a_{62}q_2 + a_{63}q_3 + a_{64}q_4 + \cdots + a_{66}q_6 + e_6 = q_6
\]

The prime applied to each variable indicates "future" value.

The power of our assumption that the technology of production is constant is now clear. With it, we have reduced the number of unknowns from 48 to six, the \( q \)'s, and can proceed to solve the system and thus to determine the outputs of industries in our economy in the future.

Compressed, the system is expressed as

\[
q_i' = \Sigma_j a_{ij} q_j' + e_i'.
\]

(5-6)

A full explanation of the solution to this system can now be easily expressed in matrix algebra and, in this case, is analogous to the one in simple algebra used with economic-base models. We wish to solve the following equation for
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\( q \), where \( q \) is a column vector of the \( q_i \) in the equation system above, \( A \) is a matrix of the \( a_{ij} \)'s, and \( e \) is a column vector of the \( e_i \)'s. Ignoring the prime marks, equation 5-6 can be rewritten as:

\[
q = A * q + e \tag{5-7}
\]

Now, we subtract \( Aq \) from both sides of the equation,

\[
q - A * q = e \tag{5-8}
\]

factor \( q \) from the terms on the left,

\[
(I - A) * q = e \tag{5-9}
\]

and multiply both sides by the inverse of \( (I - A) \) to get

\[
(I - A)^{-1} * (I - A) * q = (I - A)^{-1} * e, \tag{5-10}
\]

or, since a matrix multiplied by its inverse yields an identity matrix,

\[
q = (I - A)^{-1} * e, \tag{5-11}
\]

the solution for \( q \) in terms of \( e \). Here, \( I \) is the identity matrix, which is the matrix equivalent to the number 1 and the exponent \((-I)\) shows that the parenthetic expression is inverted, or divided into another identity matrix. The term \((I - A)\) is sometimes called the "Leontief matrix" in recognition of Wassily Leontief, the originator of input-output economics; \((I-A)^{-1}\), of course, is called the "Leontief inverse." A more descriptive title is "total-requirements table."

### Table 5.3 Total requirements (with households endogenous)

<table>
<thead>
<tr>
<th>Industry</th>
<th>Extraction (1)</th>
<th>Construction (2)</th>
<th>Manufacturing (3)</th>
<th>Trade (4)</th>
<th>Services (5)</th>
<th>Household expenditures (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraction</td>
<td>1.14</td>
<td>0.03</td>
<td>0.06</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Construction</td>
<td>0.02</td>
<td>1.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.19</td>
<td>0.26</td>
<td>1.18</td>
<td>0.12</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>Trade</td>
<td>0.17</td>
<td>0.21</td>
<td>0.14</td>
<td>1.16</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td>Services</td>
<td>0.35</td>
<td>0.35</td>
<td>0.28</td>
<td>0.43</td>
<td>1.49</td>
<td>0.50</td>
</tr>
<tr>
<td>Households</td>
<td>0.69</td>
<td>0.60</td>
<td>0.52</td>
<td>0.80</td>
<td>0.75</td>
<td>1.38</td>
</tr>
</tbody>
</table>

Table 5.3 shows the total-requirements matrix for the hypothetical economy. Each entry shows the total purchases from the industry named on the left for each dollar of delivery to final demand by the industry numbered across the top. This table reports all of the purchases due to delivery to final demand. As the title shows, the table records both direct and indirect flows.

Now, let’s insert summing rows into the table and add another meaning to it. The result is shown here as Table 5.4 and renamed as a table of "industry-output multipliers". The key row is labeled "total industry outputs" and included the sum of industry outputs numbered across the top.
required for the industry named at the top to deliver one dollar’s worth of output to final demand (that is, to export this amount). Thus, in exporting a dollar’s worth of output the manufacturing sector would cause production by other industries amounting to $1.67. This seems magical if not impossible until you recall the lessons of economic-base theory. The total double-counts the values of outputs bought and rebought as materials move from one processor to another, acquiring more value each time. We will pursue this in a little more detail in the next chapter.

Since the household sector is not really an “industry” but has been included to assure that all internal flows related to production are counted, we exclude it from the summation of industry outputs. Later, we will treat entries in the household row as "household-income multipliers."

For many years, the total of both industry outputs and household incomes in this total-requirements table was discussed as an "output multiplier." This was incorrect, however, and most analysts avoid this error. The sum in Table 5.4 is labeled as "total activity," but it has no real meaning beyond noting the rippling of money through the economy.

### Table 5.4 Industry-output multipliers (with households endogenous)

<table>
<thead>
<tr>
<th>Industry</th>
<th>Extraction (1)</th>
<th>Construction (2)</th>
<th>Manufacturing (3)</th>
<th>Trade (4)</th>
<th>Services (5)</th>
<th>Household expenditures (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraction</td>
<td>1.14</td>
<td>0.03</td>
<td>0.06</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Construction</td>
<td>0.02</td>
<td>1.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.19</td>
<td>0.26</td>
<td>1.18</td>
<td>0.12</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>Trade</td>
<td>0.17</td>
<td>0.21</td>
<td>0.14</td>
<td>1.16</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td>Services</td>
<td>0.35</td>
<td>0.35</td>
<td>0.28</td>
<td>0.43</td>
<td>1.49</td>
<td>0.50</td>
</tr>
<tr>
<td>Total industry outputs</td>
<td>1.86</td>
<td>1.86</td>
<td>1.67</td>
<td>1.75</td>
<td>1.86</td>
<td>0.93</td>
</tr>
<tr>
<td>Households</td>
<td>0.69</td>
<td>0.60</td>
<td>0.52</td>
<td>0.80</td>
<td>0.75</td>
<td>1.38</td>
</tr>
<tr>
<td>Total &quot;activity&quot;</td>
<td>2.54</td>
<td>2.46</td>
<td>2.19</td>
<td>2.55</td>
<td>2.60</td>
<td>2.32</td>
</tr>
</tbody>
</table>

Now that we have developed the logic of a regional input-output model and can see that it is a means for tracing the effects on local industries of changes in the economy, let us go back and examine the effect of closing the model with respect to households. Recall that we have included the household sector as the sixth industry in the model. Under these conditions, the total-requirements table traces the flows of goods and services required to accommodate changes in final demand through all industries and through households as well. What if the household sector had been left in final demand? What if we had continued to treat it as exogenous to the system rather than endogenous?

Table 5.5 reports a total-requirements table which is based on a five-industry version of Table 5.2, the direct-
requirements matrix. Examination of the column sums in the rows entitled "total output" in each table reveals the importance of the household sector in generating new activity in the economy.

Table 5.5 Total requirements (with households exogenous)

<table>
<thead>
<tr>
<th>Industry</th>
<th>Extraction (1)</th>
<th>Construction (2)</th>
<th>Manufacturing (3)</th>
<th>Trade (4)</th>
<th>Services (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraction</td>
<td>1.13</td>
<td>0.02</td>
<td>0.05</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Construction</td>
<td>0.01</td>
<td>1.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.11</td>
<td>0.19</td>
<td>1.12</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Trade</td>
<td>0.04</td>
<td>0.10</td>
<td>0.05</td>
<td>1.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Services</td>
<td>0.10</td>
<td>0.14</td>
<td>0.09</td>
<td>0.15</td>
<td>1.23</td>
</tr>
<tr>
<td>Total outputs</td>
<td>1.40</td>
<td>1.46</td>
<td>1.32</td>
<td>1.21</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Table 5.6 compares these tables. Just including the household sector in the inverted table leads to increases in output by the processing industries (1 through 5) of 27 to 44 percent. (When we include households as an industry and count the flows through it as “industry output,” the percent increase in output rises from 66 to 111 percent of the flows based on a table excluding households.) As we shall see later in a more detailed discussion of multipliers, income flows induced by households are important to a regional input-output analysis, but we must be careful to distinguish increases in household incomes from increases in industry outputs.

Table 5.6 A comparison of output multipliers under different household assumptions

<table>
<thead>
<tr>
<th>Industry</th>
<th>Output multipliers with: households exogenous (1)</th>
<th>households endogenous (2)</th>
<th>Increase due to household inclusion (3)</th>
<th>Percent increase due to household inclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extraction</td>
<td>1.40</td>
<td>1.86</td>
<td>0.46</td>
<td>33</td>
</tr>
<tr>
<td>Construction</td>
<td>1.46</td>
<td>1.86</td>
<td>0.40</td>
<td>27</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.32</td>
<td>1.67</td>
<td>0.35</td>
<td>26</td>
</tr>
<tr>
<td>Trade</td>
<td>1.21</td>
<td>1.75</td>
<td>0.54</td>
<td>44</td>
</tr>
<tr>
<td>Services</td>
<td>1.35</td>
<td>1.86</td>
<td>0.50</td>
<td>37</td>
</tr>
</tbody>
</table>
Economic change in input-output models

Causes vs. consequences of change

An input-output model is designed to trace the effects of changes in an economy which has been represented in an input-output table. Such models show the consequences of change in terms of flows of monies through an economy and in terms of incomes generated for primary resource owners. The models themselves do not show the causes of change; these causes are exogenous to the system.

Economic change as traced through an input-output model can take two forms: (1) structural change or (2) change in final demand. Changes in the economic structure of an area can be initiated in several ways. It can be through public investment in schools, highways, public facilities, etc., or it can be through private investment in new production facilities, or it can be through changes in the marketing structure of the economy. Changes in final demand are basically changes in government expenditure patterns and changes in the demands by other areas for the goods produced in the region.

Structural change

Structural change in an input-output context can be interpreted to mean it changes in regional production coefficients." In turn, this can be interpreted as either changes in technology or changes in marketing patterns or both. Let us see what this means in terms of the direct-requirements matrix, or the A matrix of our earlier discussion. Recall that $a_{ij}$ is the proportion of total inputs purchased from industry $i$ by industry $j$ in the region. We can treat this regional production coefficient as the product of two other coefficients and write it symbolically as

$$a_{ij} = p_{ij} r_{ij}.$$ 

The "technical production coefficient," $p_{ij}$, shows the proportion of inputs purchased from industry $i$ by industry $j$ without regard to the location of industry $i$, while the "regional trade coefficient," $r_{ij}$, shows the proportion of that purchase made locally.

A change in technology, or a change in $p_{ij}$, could be illustrated by a shift from glass bottles to metal cans by the soft-drink industry. But a change in location of purchase, or a change in $r_{ij}$, would be illustrated by a shift from metal cans made in another area to metal cans produced in our region.

The above changes are couched in terms of existing industries. Another way in which change can take place is through the introduction of new plants or even new industries. The introduction of a new plant into an existing industry has the effect of changing the production and trade patterns of the aggregated industry to reflect more of the transactions specific to the detailed industry of which the new plant is a member. For example, consider the manufacturing sector of our highly aggregated five-industry model. As presented, it reflects the combination of all manufacturing activities in the region. The introduction of new plants in the transportation-equipment industry would change the combination of purchases presently made in the manufacturing sector. The same statement might be made concerning the purchase pattern displayed by the transportation equipment industry if a new aircraft-
producing plant were established (or an old one were to cease operation).

The addition of a completely new industry to the system means adding another row and column to the interindustry table to represent the new industry. This is done in a manner similar to that involved in closing the table with respect to households.

To account for structural changes which are caused by changes in technology or in marketing requires a revision of the interindustry flows table and is best accomplished when a biennial revision is made.

**Changes in final demand**

Accounting for structural changes in an input-output model requires substantial skill and familiarity with the mechanics of the model on the part of the analyst. This is not the case when accounting for the effects of changes in final demand. It can easily be accomplished with the inverse matrix, or the total-requirements table, Table 5.3 in this chapter or, more appropriately, with its detailed equivalent.

Two kinds of changes can be traced. One form is a set of long-run changes in the demands for the outputs of all industries. This set takes the form of a vector of predicted exogenous demands (the \( e' \) vector discussed above) and represents our best judgment of the export demands for the products of our industries in some later year. Using the formula

\[
q' = (I - A)^{-1}e'
\]

we can easily derive projections of the expected gross outputs (\( q' \)) of industries in the later year.

The other form which change in final demand might take is an assumed change in the final demand for the output of one industry. Say we wish to know the effect on the economy of a $100,000 change in the demand for floor coverings. We would simply go to the detailed tables and look for the column sum for the floor-covering industry in the total requirements matrix. Using the detailed table for our hypothetical economy (not shown here), this entry is 1.8094; multiplied by $100,000, it shows that these additional export sales of carpets would increase regional outputs by a total of $180,940. A look at the household row in that same column would have yielded a household-income coefficient of .4136, meaning that the additional carpet sales would have increased local household incomes by $41,360.

The example can be pursued on a more gross level by looking at Figure 4.1 and assuming a $100,000 increase in the output of the manufacturing sector. The output multiplier in manufacturing is 1.67, meaning that the $100,000 change in export demand yields $167,000 in additional business to local firms. The household-income coefficient of .52 means that household incomes increase by $52,000. The differences between these figures and those in the above paragraph show the consequences of aggregation, which conceals a substantial amount of variation in the detailed tables.

We discuss the multiplier model in more detail in the next chapter.
Illustration 5.1 The simple input-output model

Definitions or identities:
Inputs = Sum of purchases from other local industries and from final-payment sectors
\[ g_i = z_{i1} + z_{i2} + z_{i3} + v_i + m_i \]
\[ g_2 = z_{i2} + z_{i2} + z_{i3} + v_2 + m_2 \]
\[ g_3 = z_{i3} + z_{i3} + z_{i3} + v_3 + m_3 \]
or, in matrix terms,
\[ g = Z^T i + v + m \]
where \( Z^T \) is a transpose and \( i \) represents the summing vector.

Outputs = Sum of sales to other local industries and final users
\[ q_1 = z_{11} + z_{12} + z_{13} + e_1 \]
\[ q_2 = z_{21} + z_{22} + z_{23} + e_2 \]
\[ q_3 = z_{31} + z_{32} + z_{33} + e_3 \]
or, in matrix terms,
\[ q = Z i + e \]

Behavioral or technical assumptions:

Constant production coefficients
\[ p_{ij} = \frac{r_{ij}}{g_j} \]
or \[ r_{ij} = p_{ij} g_j \]

Constant regional trade coefficients
\[ r_{ij} = \frac{(z_{ij} - m_j) / r_{ij}}{Z_{ij}} \]

Equilibrium condition:
Inputs = Outputs
\[ g_i = q_i \]

Solution by substitution:
Problem: given final demands \( e \), reduce the number of unknowns to equal the number of equations.

Let \( a_{ij} = p_{ij} r_{ij} \)

Then \[ a_{ij} = \frac{(z_{ij} - m_j) / r_{ij}}{z_{ij} / q_i} = z_{ij} / q_i \] or \[ z_{ij} = a_{ij} q_i \]

Substituting into the output equations,
\[ q_1 = a_{11} q_1 + a_{12} q_2 + a_{13} q_3 + e_1 \]
\[ q_2 = a_{21} q_1 + a_{22} q_2 + a_{23} q_3 + e_2 \]
\[ q_3 = a_{31} q_1 + a_{32} q_2 + a_{33} q_3 + e_3 \]

Or, continuing in matrix terms,
\[ q = Aq + e \]
\[ q - Aq = e \]
\[ (I - A)q = e \]
\[ q = (I - A)^{-1} e \]

Output multipliers:
\[ \frac{dq}{de_j} = r_{ij}, \text{ where } r_{ij} \text{ is an element of } R = (I - A)^{-1} \]
Each of these partial output multipliers shows the change in local output \( i \) associated with a change in exports by industry \( j \). Their sum over \( i \) is the total output multiplier for industry \( j \).

Income multipliers:
\[ income_j = \text{Sum}(r_{ij} * v_{jh} / q_i), \text{ where } v_{jh} \text{ is household income.} \]
Each of these income multipliers shows the change in household income caused by a change in exports by industry \( j \).