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6 REGIONAL INPUT-OUTPUT MULTIPLIERS

Introduction

As noted in Chapter 5, a regional input-output model can be used to provide a set of economic multipliers with which to trace the effects of changes in demand on economic activity in the region. After explaining more graphically the multiplier concept, this chapter formulates employment, household-income, and government-income multipliers and presents examples.

The multiplier concept

Input-output models are most commonly used to trace individual changes in final demand through the economy over short periods of time. In this function, they are called impact models, or multiplier models. Let us now look more closely at these models. First, we take an intuitive approach to understanding the meaning of the total-requirements table, illustrating the multiplier effect with a two-dimensional chart. Then we solve the model using the formula for the sum of an infinite series and present the results in three dimensions.

An intuitive explanation

A total-requirements table for the aggregated six-industry model was presented in Table 5.3 for the previous chapter. This table shows the direct, indirect, and induced changes in industry outputs required to deliver units to final demand. A table of this type is the basic element used in estimating multipliers.

<table>
<thead>
<tr>
<th>Table 6.1 Hypothetical direct-requirements table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
</tr>
<tr>
<td>Extraction</td>
</tr>
<tr>
<td>Construction</td>
</tr>
<tr>
<td>Manufacturing</td>
</tr>
<tr>
<td>Trade</td>
</tr>
<tr>
<td>Services</td>
</tr>
<tr>
<td>Households</td>
</tr>
</tbody>
</table>

To better understand the meaning of a total-requirements table and the multipliers derived from it, let us go back and trace the flow of outputs induced by a $100 purchase from the manufacturing sector through the direct-requirements table (repeated as Table 6.1). The result of this step-by-step tracing is illustrated in Figure 6.1 and reported in Table 6.2. First, $100 enters the state economy through manufacturing. (My convention is to label this initial round as “round zero” -- this is the “insertion” before the rounds...
of circulation begin. It is also consistent with the exponents in the solution based on the sum of an infinite series.) To produce output worth $100, firms in manufacturing purchase inputs from other industries in the economy. According to column 3 in the direct-requirements table, $4.20 goes to agriculture and mining, $0.30 goes to construction, an additional $9.80 goes to other firms in manufacturing, $3.70 goes to trade, $6.10 goes to services, and $26.10 is paid to households in wages and salaries (This is round one.). Capacity permitting, each of these industries must expand its output to accommodate this additional production load.

Thus, in producing additional output valued at $4.20, firms in agriculture and mining buy output worth $0.46 (10.9% of $4.20) from others in this sector, $0.03 (0.8% of $4.20) from construction, $0.36 (8.5% of $4.20) from manufacturing, and so on, for a total of $2.72. At the same time, each of the other industries is purchasing the additional inputs required to produce the output requested of them. The results are summarized in Figure 6.1 as the second round of purchases. Other purchases follow in succeeding rounds, each smaller as money flows out of the interindustry sector into the hands of the owners of primary inputs (excluding labor), into government coffers, and for the purchase of imported materials.

This chain of purchases continues for all industries until the economy is again in equilibrium. The initial $100 purchase from manufacturing has led to money flows throughout the regional economy valued at $219, as shown in Table 5.4 in the previous chapter. Notice that if the value of additional output is summed through round six, most of the effect of the initial purchase has already been realized: as seen in Table 6.2, $214 has been spent at this point. The total-requirements table just...
counts the rounds of spending to infinity and adds them up. The total appears in Table 5.4 in the previous chapter as the column sum for manufacturing, and is labeled "total activity." The column itself has elements which show the output impact of the initial expenditure on each industry in the system.

Now, before we go on to explore mathematically this alternative approach to solution by inversion, let us reduce the "activity effect" to a "multiplier effect" focusing on industries. This reduction is accomplished simply by excluding the household row in the table of rounds of spending. Now we count only the effects on industries. This leads to Figure 6.2. The striking thing about the contrast of these two effects is the importance of the household in a regional economy. (The same relative pattern occurs in all regional models which I have encountered.)

The iterative approach

This intuitive explanation may be expressed in a more concise mathematical form as a “solution by iterations.” This process uses the sum of an infinite series to approximate the solution to the economic model. In matrix algebra, the process is:

\[ T = IX + AX + A^2 X + A^3 X + \ldots + A^n X \]

Here, \( T \) is the solution, a column vector of changes in each industry’s outputs caused by the changes represented in the column vector \( X \). \( A \) is the direct-requirements matrix. \( n \) is the number of rounds required to produce results approximating those obtained when \( n \) approaches infinity. (For this and most other cases, \( n = 6 \) is high enough to capture over 97 percent of the flows; eight rounds capture over 99 percent.)

Table 6.2 The multiplier effect of $100 in manufacturing output traced in rounds of spending through the hypothetical economy

<table>
<thead>
<tr>
<th>Industry</th>
<th>-----Round of spending-----</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0)</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>Extraction</td>
<td>(1)</td>
<td>0.00</td>
<td>4.23</td>
<td>1.09</td>
<td>0.40</td>
<td>0.20</td>
</tr>
<tr>
<td>Construction</td>
<td>(2)</td>
<td>0.00</td>
<td>0.30</td>
<td>0.23</td>
<td>0.28</td>
<td>0.14</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>(3)</td>
<td>100.00</td>
<td>9.82</td>
<td>3.74</td>
<td>1.60</td>
<td>1.05</td>
</tr>
<tr>
<td>Trade</td>
<td>(4)</td>
<td>0.00</td>
<td>3.67</td>
<td>4.93</td>
<td>1.87</td>
<td>1.48</td>
</tr>
<tr>
<td>Services</td>
<td>(5)</td>
<td>0.00</td>
<td>6.09</td>
<td>9.39</td>
<td>4.84</td>
<td>3.29</td>
</tr>
<tr>
<td>Total industry outputs</td>
<td>100.00</td>
<td>24.11</td>
<td>19.38</td>
<td>8.98</td>
<td>6.15</td>
<td>3.30</td>
</tr>
<tr>
<td>Households</td>
<td>(6)</td>
<td>0.00</td>
<td>26.10</td>
<td>8.60</td>
<td>7.72</td>
<td>3.57</td>
</tr>
<tr>
<td>Total &quot;activity&quot;</td>
<td>100.00</td>
<td>50.21</td>
<td>27.97</td>
<td>16.71</td>
<td>9.72</td>
<td>5.77</td>
</tr>
</tbody>
</table>
The multiplier-effects, or "rounds-of-spending," table (Table 6.2) lays out the results of applying this equation to the problem of tracing direct expenditures. In this table, the total column is $T$. Round 0 is $IX$, the initial expenditures to be traced ($I$ is the identity matrix, which is operationally equivalent to the number 1 in ordinary algebra). Round 1 is $AX$, the result of multiplying the matrix $A$ by the vector $X$. It answers the question: what are the outputs required of each supplier to produce the goods and services purchased in the initial change under study? Round 2 is $A^2X$, which is the same as $AAX$, or $A(AX)$; it is the result of multiplying the matrix $A$ by Round 1 (or $AX$). It answers for Round 2 the same question as was applied to Round 1: what are the outputs required of each supplier to produce the goods and services purchased in Round 1 of this chain of events? Round 3 is $A^3X$, which is the same as $A(A^2X)$; and the story repeats, round after round. Each of columns 1 through 6 in the multiplier-effects table represents a term in the continuing but diminishing chain of expenditures on the right side of the equation.
The tracing of these rounds in Table 6.2 can also be presented in three dimensions with a spreadsheet program (I used Excel). Figure 6.3 shows once again the multiplier effects (now it might be proper to join the newspapers in discussing the "ripple effect") of a $100 export sale of manufacturing goods by our economy. Notice the large flow through households, especially in round 1, and note how quickly circulation falls off. This economy is pretty leaky!

This process has several advantages to recommend it over the inversion solution. The analyst builds a direct impact vector (Round 0) through survey or assumption and then sets the iterative process in motion on a desktop computer. He can see clearly the paths taken by indirect rounds of spending and can perform a check both on his direct impact vector and on the input-output model itself. Further, he has the material available to make a graphic presentation of his results which is easily understood by a reader.

Before the advent of microcomputers, the typical input-output study included lengthy sets of inverse matrices and multiplier tables in which the analyst
looked up multipliers which applied to his problem. These numbers were then applied blindly and with faith.

**Multiplier transformations**

Now let us go back to the solution of the input-output model by inversion as presented in the previous chapter. This has the advantage of permitting us to generalize and produce the various kinds of multipliers possible in an input-output system.

**Output multipliers**

The first thing to note about Table 6.3 is that the elements of the inverse are now relabeled as "partial output multipliers." That is, in multiplier terminology, they are to be interpreted as stating the total impact on a row industry's output of one dollar brought into the economy by a column industry. The sums of these columns are output multipliers.

Note that we report three kinds of output multipliers. The first is simply 1.0, the direct sales by the industry in question, which may be called the "direct output multiplier" (sort of obvious, but included just for parallelism).

The second is a total "industrial activity multiplier", which is the sum for each column industry of the vector of partial output multipliers multiplied by direct output impacts (the 1's). To avoid considering the household as a true "industry", we violate the principles of vector multiplication by summing over industries (five rows) and not over industries and households (six rows).

\[ QMULT_j = \sum_i INV_{ij} \]

where \( i \) counts through the number of industries.

The third output multiplier reported in Table 6.3 is sometimes called a "gross output multiplier." It is the sum over both industries and the household row of a column in the inverse matrix. To emphasize its mixed origin we label it as an "industry and household multiplier" and herewith declare it as improper to use. It mixes double-counted outputs and final incomes and has no meaning.

Although we have arrived at this point with little mention of earlier practice in input-output analysis, note that it was common practice several decades ago to produce a "Type 1" model, with households excluded from the A matrix, and a "Type 2" model, with households included. Output multipliers were appropriate with the Type 1 models and inappropriate with the Type 2 models. Analysts frequently became confused.

Now, almost all regional input-output analysts have abandoned Type 1 models and concentrate on the augmented models described here. If output multipliers are to have any meaning, they should be based on industrial activity alone.

The more appropriate multipliers, however, are those transformed into terms of employment and final incomes. They represent jobs and incomes to people, two important measures of the importance of industrial activities.
Table 6.3 Output, employment, income, and local and state government revenue multipliers for hypothetical economy

<table>
<thead>
<tr>
<th>Industry title</th>
<th>Agric, mining</th>
<th>Constr.</th>
<th>Manufacturing</th>
<th>Trade</th>
<th>Service</th>
<th>Households</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Partial Output Multipliers (elements of inverse)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture, mining</td>
<td>1.139</td>
<td>0.033</td>
<td>0.062</td>
<td>0.015</td>
<td>0.023</td>
<td>0.021</td>
</tr>
<tr>
<td>Construction</td>
<td>2.019</td>
<td>1.011</td>
<td>0.012</td>
<td>0.015</td>
<td>0.040</td>
<td>0.014</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>3.184</td>
<td>0.253</td>
<td>1.173</td>
<td>0.092</td>
<td>0.128</td>
<td>0.146</td>
</tr>
<tr>
<td>Trade</td>
<td>4.165</td>
<td>0.207</td>
<td>0.138</td>
<td>1.159</td>
<td>0.166</td>
<td>0.245</td>
</tr>
<tr>
<td>Service</td>
<td>5.346</td>
<td>0.351</td>
<td>0.280</td>
<td>0.428</td>
<td>1.494</td>
<td>0.498</td>
</tr>
<tr>
<td>Households</td>
<td>6.684</td>
<td>0.593</td>
<td>0.516</td>
<td>0.785</td>
<td>0.745</td>
<td>1.381</td>
</tr>
</tbody>
</table>

**Output multipliers**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct output</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Industrial activity</td>
<td>1.853</td>
<td>1.855</td>
<td>1.664</td>
<td>1.708</td>
<td>1.850</td>
<td>0.924</td>
</tr>
<tr>
<td>Ind &amp; households</td>
<td>2.538</td>
<td>2.448</td>
<td>2.180</td>
<td>2.494</td>
<td>2.595</td>
<td>2.305</td>
</tr>
</tbody>
</table>

**Income multipliers**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct income</td>
<td>0.355</td>
<td>0.264</td>
<td>0.261</td>
<td>0.495</td>
<td>0.406</td>
<td>0.006</td>
</tr>
<tr>
<td>Total income</td>
<td>0.684</td>
<td>0.593</td>
<td>0.516</td>
<td>0.785</td>
<td>0.745</td>
<td>0.381</td>
</tr>
</tbody>
</table>

**Employment (per $ million)**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct employment</td>
<td>30</td>
<td>12</td>
<td>16</td>
<td>30</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>Total employment</td>
<td>54</td>
<td>35</td>
<td>34</td>
<td>51</td>
<td>49</td>
<td>30</td>
</tr>
</tbody>
</table>

**Local government**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct revenue</td>
<td>0.020</td>
<td>0.010</td>
<td>0.009</td>
<td>0.014</td>
<td>0.024</td>
<td>0.023</td>
</tr>
<tr>
<td>Total revenue</td>
<td>0.051</td>
<td>0.038</td>
<td>0.032</td>
<td>0.046</td>
<td>0.057</td>
<td>0.049</td>
</tr>
</tbody>
</table>

**State government**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct revenue</td>
<td>0.001</td>
<td>0.004</td>
<td>0.008</td>
<td>0.120</td>
<td>0.016</td>
<td>0.021</td>
</tr>
<tr>
<td>Total revenue</td>
<td>0.042</td>
<td>0.049</td>
<td>0.041</td>
<td>0.163</td>
<td>0.061</td>
<td>0.068</td>
</tr>
</tbody>
</table>

**Employment multipliers**

The next question is how to estimate the impact on employment of new export activity. To answer this question, we must transform output impacts into employment impact. We use employment/output ratios for this.

To keep the numbers reasonable, we denominate the ratio in employees per million dollars of output. Total employment multipliers for an industry, then, are produced by multiplying the row vector of direct employment multipliers (which are the employment/output ratios) by the appropriate column vector in the inverse matrix. The formula for the employment multiplier for industry \( j \) can be stated as:

\[
EMULT_j = \sum (n_i / q_i) \times INV_j
\]

WASchaffer 8/19 Revised 3/25/2010
and can be restated in matrix terminology for all industries as

$$EMULT = n\hat{q}^{-1}(I - A)^{-1}.$$  

where $n$ is now a vector of industry employment, $\hat{q}$ is a diagonal matrix reflecting the vector $q$, and the other symbols are as defined previously.

Note that we have approximated current employee/output ratios to avoid results you would consider outlandish if we had used the original 1970 ratios. In fact, application of an input-output model which is usually a couple of years old (almost by definition) to a current or future statement of impact requires that we develop current or future estimates of employment/output ratios.

**Income multipliers**

Transformation from output terms to income terms is similar. Here, the direct income multiplier is simply the ratio of household income to output and is taken directly from the household row of the direct requirements matrix. The total income multiplier is produced by converting total changes in output to total changes in income using these ratios.

Symbolically, the household-income multiplier is written as a transformation almost identical to that used for the employment multiplier:

$$HMULT_i = \Sigma_j (h_i/q_i) * INV_{ij}$$

Here, $h_i$ represents the household income of industry $i$.

Income multipliers are most useful compared to output multipliers. Output multipliers are "gross" in that they double-count transactions. One of the problems with economic-base multipliers was this double-counting, which led naive audiences to believe that we could magically get more stuff out of an economy than we put in, and it led sophisticated audiences into disbelief in the analyst's results. Income multipliers report the money that drops into the hands of owners of resources as income. In this case, the owners happen to be workers.

**Government-income multipliers**

The same process can be applied to government incomes, simply by substituting "local-government revenue" or "state-government revenue" for "household income" in the transformation formula. The resulting multipliers are relatively low because the revenue/output ratios are low.

Government-income multipliers suffer in interpretation because, while we generally consider governments as owners of resources and so treat them as we do households, they are in fact organized as businesses, and we hope they produce outputs in response to their incomes. This anomaly means that they should be interpreted as yielding excess revenue in some short-run period (in which excess capacity exists in streets, protection, utilities, etc.), but, in the long run, the piper must be paid and new capacity must be financed. Interpretation depends on your time horizon.
Appendix 1 Multiplier concepts, names, and interpretations

Introduction

This appendix examines the development of input-output multiplier concepts and the evolution of their names. It is included to show the evolution of the concepts over the last several decades. I will restrict this statement to regional input-output models mostly in North America and to multipliers commonly used in impact analysis, pointing out some preferred alternatives.¹

It is traditional to start such an exposition with salutes both to R. F. Kahn, who introduced the employment multiplier to describe the effect upon unemployment of a net increase in home investment (Kahn 1931), and to John Maynard Keynes, who elaborated on and expanded the concept as an income multiplier in support of his arguments on national income and employment (Keynes 1936). These are familiar beginnings for economics students.

While the concept of economic-base analysis had been at play in geography and planning for a couple of decades, the economic-base multiplier did not seriously enter the literature in economics until 1950, with estimation of an employment multiplier for Los Angeles County by George H. Hildebrand and Arthur Mace, Jr. (Hildebrand and Mace do credit M. C. Daly, writing in the Economic Journal in 1940 for their approach. (Hildebrand and Mace 1950)) From this start, economic-base multipliers thrived in regional economics. Theodore Lane provides an excellent review of economic-base industry into its last great decade (Lane 1966).

The early literature on input-output analysis contained relatively few references to multipliers, although the inverse or interdependence matrix was well known and used with national tables. Essentially, it was left to regional economists to develop input-output multipliers. The serious literature on regional input-output multipliers starts with Frederick T. Moore in a now-classic article on "Regional Economic Reaction Paths" in 1955 (Moore 1955). And this is where I begin.

Income multipliers

Origins of Types I and II income multipliers

Reporting on models of California and Utah developed in association with James W. Petersen, Moore defines "...the 'simple' income multiplier as the ratio of the direct-plus-indirect income effects to the direct alone" (Moore and Petersen 1955). His formula is well-known:

\[
M_k = \frac{\sum_j a_{hj} d_{jk}}{a_{hk}}
\]

(1)

where \( M_k \) is the simple income multiplier for industry \( k \), \( a_{hj} \) is the household income coefficient for industry \( j \), and \( d_{jk} \) is the element in the inverse showing direct and indirect purchases from industry \( j \) associated with a unit exogenous sale by industry \( k \).

Moore explains his caution in using the term "simple" with a note that the Utah study reports multipliers based on an inverse derived from a coefficient matrix closed with respect to households (but he fails to use an adjective in

¹ This appendix is based on a paper presented at the North American meetings of the Regional Science Association in 1990.
describing these other multipliers). In addition, he discusses "... one more type of relationship: the effects upon investment induced by the changes in income and consumption. These effects he calls "accelerator coefficients," and he calculates them with a modification of the numerator of his simple income multiplier:

\[ R_k = \sum_j b_j d_{jk} \]  

(2)

\( R_k \) is the accelerator coefficient, and \( b_j \) is the capital-output ratio for industry \( j \). This coefficient has apparently disappeared from the literature, probably due to data deficiencies.

Moore and Petersen report an interindustry model of Utah which remains a monument to scholarship. They go beyond Leontief's earlier-described "balanced regional model" (Leontief 1953) to develop a transactions table with pure regional transactions, albeit transactions estimated with what Schaffer and Chu later (1969) called the "supply-demand pool" technique, or the commodity-balance approach.

They proceed with development of three sets of income multipliers. Interestingly, especially given the problems we have faced over the years in terminology, they give these multipliers no specific names. The first income multiplier was the one named earlier by Moore as the "simple income multiplier." Its numerator was "income reactions to changes in demand," or the direct and indirect income effects of a unit change in final demand. The second was derived from "income reactions including induced via homogeneous consumption functions," or the direct, indirect, and induced income effects of change. The third multiplier was based on "income reactions including induced via nonhomogeneous consumption functions. This third multiplier was computed using the marginal coefficients in consumption functions estimated from national data for only three aggregations of the seven-industry model used to illustrate the study.

Moore and Petersen felt that the induced effect is overstated by the homogeneous consumption functions implied by the closure of the model with respect to households.

Werner Z. Hirsch, in his model of St. Louis, moved beyond Moore and Petersen in three ways: he built the model from company records, he prepared a detailed exports matrix, and he treated the household and local government sectors as endogenous parts of the economy (Hirsch 1959). But he remained consistent in his treatment of income multipliers, although he developed only two sets and gave them parenthetical names. Moore's simple income multiplier became "multiplier (Model I)" and his second multiplier became "multiplier (Model II)."

At this point, little attention had been paid to input-output multipliers in the literature. In his *Methods of Regional Analysis*, Walter Isard, pays scant attention to them, seven pages out of over 700, devoted almost exclusively to the income and employment multipliers of the Utah study (Isard 1960). Hollis B. Chenery and Paul G. Clark, in their *Interindustry Economics*, long the comprehensive reference in the field, cite the term "multiplier" only twice in their index (Chenery and Clark 1959). Chenery and Clark do build inverses for Model I and Model II tables, as did Hirsch, with the Model II tables including households as endogenous.
But in 1965, William H. Miernyk devoted 14 pages of over 150 to multiplier analysis, and 8 of these to income multipliers. Simple, clear, and easily understood, Miernyk's *Elements of Input-Output Analysis* (Miernyk 1965) became a virtual bible to budding regional input-output analysts. Interest in regional input-output studies was intensifying in the sixties, and with this interest came a need to present some of the tools which might help others in using models produced at substantial expense and effort.

Miernyk based his table illustrating "income interactions" on a similar table from Hirsch. But he changed column headings describing multipliers for models I and II to be "Type I multiplier" and "Type II multiplier." To my knowledge, this is the beginning of the line for these names, which are still in use today. My suspicions are confirmed by P. Smith and W. I. Morrison, who calculated income multipliers "... using methods developed by Hirsch and discussed further by Miernyk, ... described (after Miernyk) as type 1 and type 2 income multipliers" (Smith and Morrison 1974, p. 57).

**Type III multipliers**

Moore and Petersen as well as Hirsch developed crude consumption functions for sets of industries in an attempt to reduce the impact of average propensities to consume (as expressed in a household coefficients column) on the induced income effect included in their type II multipliers. But the resulting third set of multipliers was unnamed. Miernyk obliged in his Boulder study by labeling these multipliers as "type III multipliers." Miernyk calculated consumption functions in Boulder and applied estimated marginal propensities to consume instead of household coefficients in computing income multipliers. As he expected, the resulting type III multipliers were lower than type II multipliers by slightly more than 10 percent. (Miernyk et al. 1967)

A second definition of type III multipliers is in use in Canada in the Atlantic Provinces and Nova Scotia (See Jordan and Polenske 1988 for a clear summary.). Following the lead of Kari Levitt and a team at Statistics Canada, I used three levels of closure to develop output, income, and employment multipliers for the 1974, 1979, and 1984 Nova Scotia input-output models (DPA Group Inc. and Schaffer 1989). Model I is the traditional open model, yielding direct and indirect effects; model II is closed with respect to households, yielding direct, indirect, and induced effects; model III is closed with respect to local and provincial government sectors as well, yielding direct, indirect, and extended effects. We devised the term "extended effects" to include effects induced both by households and by local and provincial governments.

This treatment of type III multipliers as reflective of greater closure seems a better use of the numbering system.

**Household-income multipliers**

Income multipliers are the most troublesome of multipliers. When we documented the Hawaii input-output model for 1967 the problems became obvious (Schaffer et al. 1972). Because it was by then traditional to show income and output multipliers, we did so, designating them as "simple" and "total," feeling these terms more descriptive than "type I" and "type II." (The "simple" came from the income multiplier in Moore and Petersen; the
"total" came from their employment multiplier.) But we could not recall any instance in which the Department of Planning and Economic Development or any other agency had been asked to show the total effect on household incomes of a specified direct income change due to exogenous forces on an industry. People asked about the impact of new plant investment (a problem for which no convenient multipliers have been constructed) and about the impact of export sales, but none would ask about the effect of a payroll increase. So we added an "income coefficient," showing the total income effect associated with a unit change in export sales. The format of the table was Hirsch's (1959, p. 366), with a total column added.

In documenting the 1970 Georgia input-output model, (Schaffer 1976), the traditional definition became "... awkward and inconsistent with the approach taken in most interpretations. Therefore, we ... defined the 'household-income multiplier' for industry \( j \) to be the addition to household incomes in the economy due to a one-unit increase in final demand for the output of industry \( j \)." The footnote explaining this move is a clear summary of the twisted path described thus far:

The notion of an income multiplier as first introduced to the regional literature by Moore and Petersen (1955) was used to describe additional household income attributable to unit change in household income in the industry in question. This definition is far removed from the exogenous changes which precipitate additional income changes, and it forces its user to unnecessary trouble in determining, e.g., the importance of an industry to its community. We have therefore switched ... Although a similar concept appeared in Moore and Petersen, ... our point is that the label is confusing. We first saw this concept discussed as 'income coefficients' in the Mississippi study by Carden and Whittington (1964) and used the term to distinguish our multiplier from the one seen in the literature as late as 1973. (p. 67)

Davis (1968, p. 33) makes the importance of this distinction clear. Two old-style income multipliers may have exactly the same values but may differ substantially in terms of income change per unit change in final demand. He suggests that an emphasis on absolute changes ... rather than on relative changes, as in Moore and Petersen, be made to avoid the confusion. The caution was also expressed by Miernyk (1967, p. 101) in his Boulder study, which reported old-style income multipliers.

But despite these problems, the old-style income multipliers live on. Two not-too-distant uses are in the Delaware model produced by Sharon Brucker and Steve Hastings (1984) and in the 1984 Nova Scotia Study, where the term 'income-generated multiplier' substitutes for the above-recommended 'household-income multiplier' and 'income multipliers' are defined in the original way.

Other income multipliers

It became obvious early that other multipliers reflecting the total impact of changes in final demand on incomes could be generated.

The documentation of the 1963 Washington input-output (Bourque and others 1967) reports only one multiplier, the regional income multiplier, which includes the change in total state income per $100,000 change in final demand. The 1972 Washington study (Bourque and Conway 1977), in one of the clearest expositions of multiplier development yet written, defines type I and type II value added multipliers, along with labor-income multipliers, and jobs multipliers. In each case, the multiplier is calculated as "... a simple transformation of the output multipliers given in the inverse matrix ... ." Thus:
\begin{equation}
M_k = \sum_i v_{ki} b_{ij} \sum v_{kj} b_{ij}
\end{equation}

where \( M_k \) is the income multiplier for the \( k^{th} \) factor income, \( v_{ki} \) is the input coefficient for the \( k^{th} \) factor in industry \( i \), and \( b_{ij} \) is the inverse coefficient.

This view of the multiplier as an operator with which to transform changes in industry output into impacts on incomes of factors of production was well expressed by Hays B. Gamble and David L. Raphael (1966). In a survey-based model of Clinton County, Pennsylvania, they develop "residual income multipliers" by inverting a coefficient matrix including vectors for the nonprofit sector, three local government sectors, and four household sectors. Then by summing elements in these rows of the interdependency matrix (inverse) for each industry, they produced household multipliers, local government multipliers, and nonprofit multipliers.

In both the Nova Scotia studies and the Georgia model, I produce government-income multipliers for local governments and for provincial or state governments.

In all these cases, the multipliers can be unambiguously interpreted as changes in incomes per change in exogenous demand. It is only the income multiplier associated with households that is ambiguous.

**Employment multipliers**

Employment multipliers were developed by Moore and Petersen at the same time as and analogous to their income multipliers. They estimated a set of aggregated employment-production relationships and used the coefficients showing change in employment per change in output as the \( a \)'s in equation (1), transforming the multiplier from dollar terms into employment terms. Their multipliers relate total employment change to direct employment change.

Both Hirsch and Miernyk repeat this definition. In his Boulder study, Miernyk built "type III" employment multipliers as well, with a complete set of employment functions. Later, in his West Virginia Study, Miernyk apparently used simple employment-output ratios to effect the transformation from output terms to employment terms.

The first employment multipliers produced in the Washington series were for 1972. Bourque and Conway unwaveringly maintain their devotion to defining multipliers denominated with final demand changes, producing type I and type II versions to reflect levels of closure.

In Georgia, I also denominated my simple and total employment multipliers with final demand changes. In Nova Scotia, we produced both output-based employment multipliers and employment-based employment multipliers for models with three levels of closure, parallel to our other multipliers. This was largely because of a feeling that, in inflationary times, the shelf life of output-based employment multipliers was much lower than that of employment-based ones.

**Output multipliers**

Output multipliers are an obvious starting point for multiplier analyses. They are simply the column sums of the industry part of an inverse matrix. These columns are the essential ingredients in the other multiplier calculations. Yet, output multipliers were neither
mentioned nor alluded to by either Moore and Petersen or Hirsch or Miernyk.

Floyd Harmston in his 1962 Wyoming study used a “business and industry multiplier” which was a type I output multiplier under another name (Harmston 1962). I used output multipliers in both Georgia and Nova Scotia studies. But, after a little note from Rod Jensen, Australia’s leading authority on input-output analysis, commenting on an early draft, I made explicit a warning that output multipliers should be used only as general indicators of industrial activity. While they represent the essence of interindustry models, the flows of intermediate transactions are not our basic interests in summary multipliers. Effects on income and employment are our primary concerns. Nevertheless, these by-products of useful multiplier production are common statements in many studies.

In explaining why they did not produce aggregate output multipliers for the 1972 Washington model, Bourque and Conway express this caution beautifully:

... we have not specified aggregate output multipliers. Although the output multipliers given by the elements of the inverse matrix are at the root of the multipliers ...[calculated] ... . it is not very meaningful to sum these elements into aggregate output multipliers for each industry ... . In other words, given the concepts that lie behind each output measure, it does not make much sense, economically speaking, to combine, say, the shipments of pulp mills with the margins of the insurance industry into an aggregate transactions measure. Furthermore, users of the Washington tables in the past have sometimes employed aggregate output multipliers inappropriately, in at least one case confusing them with income multipliers. For these reasons, we have chosen not to present aggregate output multipliers. (Bourque and Conway 1977, p. 48)

Observations and summary

These comments are consistent with conventions used in a recent comprehensive compendium on input-output analysis by Ronald E. Miller and Peter G. Blair (1985). Multipliers have admittedly become a larger question than for earlier writers. Miller and Blair devote over ten percent of their book to regional, interregional, multiregional, and interrelation multipliers. (Only five percent is devoted to basic regional multipliers.)

They resolve my concern regarding the denominating of income and employment multipliers by relegating the terms "type I" and "type II" exclusively to multipliers with denominators in the same terms as the numerators. They then use "simple" to identify multipliers based on pure industry transactions tables and "total" for multipliers based on tables closed with respect to households. “Income” multipliers show changes in income per unit change in final demand and “employment” multipliers are expressed per unit change in final demand as well.

These statements and the explanations accompanying them are helpful indeed. But, an appropriate conclusion of this review is that analytic purposes determine the terminology to use. The problem which we might perceive comes from overall study reports which suggest through terminology the impacts which might be examined through regional input-output models. When particular questions are pursued, explanations should eliminate the ambiguities in impact interpretations.
Further extensions

The point of view taken above can be described as that of an old-line traditionalist. More multipliers continue to evolve and recirculate. I should mention two of them.

One is income-distribution multipliers. To my knowledge, efforts to develop income-distribution multipliers started with Roland Artle in his studies of the Stockholm economy (Artle 1965) and in his efforts in the early sixties to estimate such multipliers while working with a research team in Hawaii. While he conceptualized a system with the household row divided into several income classes and with the household expenditure column similarly divided, the effort failed due to complex data problems.

An effort which has yielded more fruit is that of Kenichi Miyazawa documented in several articles in the sixties and seventies and in his book (Miyazawa 1976). His “intersectoral income multipliers” have been used on numerous occasions in the last two decades. Since such studies are beyond the realm of normal economic impact analyses, we do not treat them further.

The second multiplier system that has been resurrected is the “SAM-based multiplier.” Social Accounting Matrices (SAMs) evolved to a high level in the sixties in several streams of literature. One is the work of Richard A. Stone (leading to his Nobel Laureate in Economics) and extended at great detail in the United Nations social accounting literature. Another is the work at Statistics Canada as exemplified in the most exhaustive theoretical and empirical analysis of social accounts by Kari Levitt for the Atlantic Provinces (Levitt 1975a; 1975b). A third outstanding example of social accounting was executed by the late Jerald R. Barnard at the University of Iowa (Barnard 1967).

Each of these streams resulted in interindustry tables augmented by social accounts documenting incomes in various sectors of the economy (detailed income and product accounts of which the aggregated regional accounting table in Chapter 3 is a simple example) and yielding multipliers for the various sectors.

A more recent example of SAM multiplier models is the one in current use by IMPLAN, the modeling system developed by MIG, Inc. (Minnesota IMPLAN Group Inc. 1997)

A final comment is simply a caution. The proliferation of multipliers over the years and the complexity of extensions to input-output analysis can lead to substantial confusion among even knowledgeable analysts. Any multiplier analysis intended for a broad audience (or even an important decision-making audience) should be reduced to the simplest possible terms but should be sufficiently documented to permit some evaluation of its conclusions.
7 INTERREGIONAL MODELS

Now we can easily combine the economic-base and interindustry concepts to yield a quick feeling for multi-regional models. While we stop with this introduction, extensions to include transactions between industries in different regions are immediately obvious. In fact, a considerable literature has developed on interregional interindustry models and efforts have even been made to relate such models to “other worlds” such as the environment delineating the interactions of the human economy with the highly interactive elements of our land, sea, and air “economies.”

Interregional economic-base models

In the following summary models, I have left implicit the definitions and identities which we labored over before. The common glossary is as follows:

Y: income
E: income-related expenditures
E_i: income-related expenditures in region i
E_{ij}: income-related expenditures in region i by residents of region j
A: autonomous expenditures
e: marginal propensity to spend, dE(Y)/dY=e
X: exports
M: imports
m: marginal propensity to import, dM(Y)/dY=m

i: subscript for region (A subscript of 0 is for “autonomous value.”)

Review of one-region models

The one-region models are the Keynesian and economic-base models. Their distinguishing characteristic is that the value of income is determined by some autonomous activity out there. It may be investor behavior, as in the Keynesian model, or it may be purchases by consumers outside the region, as in the economic-base model.

Closed economy, no external trade

Assumptions:
E = E(Y) = eY
A = A_0

Equilibrium condition:
Y = E + A

Solution:
Y = eY + A_0
Y = [1/(1 - e)] A_0

Multiplier:
dY/da = 1/(1-e)

Open economy, with trade leakage

Assumptions:
E_i = E_i(Y_i) = e_i Y_i
X_i = X_0
M_i = M_i(Y_i) = m_i Y_i
A_i = A_0

Equilibrium condition:
Y_i = E_i + A_i + X_i - M_i

Solution:
Y_i = e_i Y_i + A_0 + X_0 - m_i Y_i
Y_i = (A_0 + X_0)* 1/(1 - (e_i - m_i))
\[ \frac{dY_i}{dX_0} = \frac{dY_i}{dA_0} = 1/(1-(e_1 - m_i)) \]

**Two-region model with interregional trade**

What has been missing all along has been interaction between regions. Obviously this takes place in the real world; otherwise, there would be no regions. Exports by one region have to be imports into another economy.

**Partial solution, two-region interregional model**

The system can more easily be presented by laying out the system in the same form as above but stopping before the algebra requires the theory of determinants, which has either been forgotten or ignored by most of us. We will limit the statement to region one alone in this section.

**Assumptions:**

\[ E_i = E_i(Y_i) = e_i Y_i \]
\[ X_i = M_2 = M_2(Y_2) = m_2 Y_2 \]
\[ M_i = M_i(Y_i) = m_i Y_i \]
\[ A_i = A_0 \]

**Equilibrium condition:**

\[ Y_i = E_i + X_i - M_i + A_i \]

**Solution:**

\[ Y_i = e_i Y_i - m_i Y_i + m_2 Y_2 + A_0 \]
\[ dY_i = e_i dY_i - m_i dY_i + m_2 (dY_2/dY_i) dY_i + dA_0 \]
\[ dY_i = e_i dY_i - m_i dY_i + m_2 (dY_2/dY_i) dY_i (dM_i/dX_2) + dA_0 \]
\[ dY_i = e_i dY_i - m_i dY_i + m_2 m_1 (dY_2/dX_2) dY_i + dA_0 \]

But this line of attack becomes unnecessarily complicated. We can't easily reduce the interactive term without knowing the multiplier for the other region \((dY_2/dX_2)\)!

**Multiplier:**

\[ \frac{dY_i}{dA_0} = 1/(1-(e_i - m_i + m_2 m_1)) \]

\[ \frac{dY_i}{dA_0} = 1/(1-(e_i - m_i + \text{interregional effect})) \]

Let us take a more familiar tack. (This approach originated with Alan Metzler (Metzler 1950); Harry Richardson provides the restatement on which the above is based (Richardson 1969).

**Matrix solution, two-region interregional model**

This model can be more systematically expressed with the matrix algebra developed for input-output analysis. Matrix algebra permits us to immediately extend our logic from two regions as above to \(n\) regions without producing page after page of cumbersome calculations. Nevertheless, we will work with a two-region system for simplicity of discussion. We proceed as follows:

**Assumptions:**

As above, but now for both regions 1 and 2.

\[ E_i = E_i(Y_i) = e_i Y_i \]
\[ X_i = M_j = M_j(Y_j) = m_j Y_j \]

**Equilibrium conditions:**

\[ E_i + X_i - M_i + A_i = Y_i \]
\[ E_2 + X_2 - M_2 + A_2 = Y_2 \]

Rewrite to align purchases from the two regions:

\[ E_i + X_i - M_i + A_i = Y_i \]
\[ X_2 + E_2 - M_2 + A_2 = Y_2 \]

Now, observing the following definitions, we can rewrite the equation system preparatory to matrix manipulation:

\[ E_{i1} = E_i - M_i = e_i Y_i - m_i Y_i = e_{i1} Y_i \]
\[ E_{12} = M_2 = M_2(Y_2) = m_2Y_2 = e_{12}Y_2 \]
\[ E_{21} = X_2 = M_1(Y_1) = m_1Y_1 = e_{21}Y_1 \]
\[ E_{22} = E_2 - M_2 = e_2Y_2 - m_2Y_2 = e_{22}Y_2 \]

Note that the \( e \)'s now show double subscripts showing purchases from region \( i \) by region \( j \). So the system becomes

\[ E_{11} + E_{12} + A_1 = Y_1 \]
\[ E_{21} + E_{22} + A_2 = Y_2 \]

or

\[ e_{11}Y_1 + e_{12}Y_2 + A_1 = Y_1 \]
\[ e_{21}Y_1 + e_{22}Y_2 + A_2 = Y_2 \]

which translates into matrices as

\[
\begin{bmatrix}
  e_{11} & e_{12} \\
  e_{21} & e_{22}
\end{bmatrix}
\begin{bmatrix}
  Y_2 \\
  Y_2
\end{bmatrix}
+ 
\begin{bmatrix}
  A_1 \\
  A_2
\end{bmatrix} =
\begin{bmatrix}
  Y_1 \\
  Y_2
\end{bmatrix}
\]

or

\[ eY + A = Y \]

where \( e \) is the matrix of \( e_{ij} \). This solves exactly as would an input-output system:

\[ Y - eY = A \]
\[ (1 - e)Y = A \]
\[ (1 - e)^{-1}(1 - e)Y = (1 - e)^{-1}A \]
\[ Y = (1 - e)^{-1}A \]

Now, to give the elements of this inverse a designation, set

\[ R = (1 - e)^{-1} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \]

rewrite the solution as

\[ Y_1 = R_{11}A_1 + R_{12}A_2 \]
\[ Y_2 = R_{21}A_1 + R_{22}A_2 \]

The interregional multipliers are thus

\[ \frac{dY_i}{dA_j} = R_{ij} \]

for each \( i \)th row and \( j \)th column.

Now, it is a simple matter to expand the system to treat \( n \) regions. The process is identical to that used in the two-region system but with larger matrices!

**Extensions and further study**

**Interregional interindustry models**

The obvious extension is to expand the cell entries to become interindustry transactions matrices. Thus the \( E_{ij} \) entries would become \( k_iE_{ij} \), showing purchases from industry \( k \) in region \( i \) by industry \( l \) in region \( j \). where \( i=j \), the regional transactions matrix for region \( i \) is inserted, and where \( i \) is not equal \( j \), a matrix of imports is inserted.

The most commonly used multi-regional interindustry model of the United States was assembled by Professor Karen Polenske at MIT. Her model includes transactions generally for 50 industries in each of 44 states or sets of states. (Polenske 1980)

**Economic-ecologic models**

Economic-ecologic models append matrices depicting interaction between sectors in the natural environment and between those sectors and the industrial ones. The concept is similar to interregional interindustry analysis but implementation becomes a harrowing experience. The dominant example of this was a complex model of the Massachusetts Bay area. (Isard et al. 1972)

Both of these extensions are discussed in detail in the comprehensive input-output reference by Ronald Miller and Peter Blair. (Miller and Blair 1985)