Heterogeneity and Chaotic Dynamics in Commodity Markets

By

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Abstract: The nonlinear testing and modeling of economic and financial time series has increased substantially in recent years, enabling us to better understand market and price behavior, risk and the formation of expectations. Such tests have also been applied to commodity market behavior, providing evidence of heteroskedasticity, chaos, long memory, cyclicity, etc. More recently the evaluation of empirical financial models suggests that chaotic structure in asset prices can result from the heterogeneity of trader's expectations. The present evaluation of futures price behavior confirms that the resulting price movements can be random, suggesting noisy chaotic behavior. The root cause of this behavior is the endogenous forces in the market, i.e. the interactions between heterogeneous investors. Thus, prices could follow a mean process that is dynamic chaotic, coupled with a variance that follows a GARCH process. Our conclusion is that models of this type could be constructed to assist in forecasting prices over short run but not over long run time periods.

1. Introduction

The nonlinear testing and modeling of economic and financial time series has increased substantially in recent years, enabling us to better deal with risk in trading spot and derivative assets. More recently such tests have been applied to commodity trading and in particular to commodity prices. These tests have included evaluations of heteroskedasticity, chaos, short and long memory, cyclicity, etc. Examples include Agnon et al. (1999), Barkoulas et al. (1997, 1999), Davidson et al. (1997), Labys et al. (1998), Decoster et al. (1992), Agnon et al. (1999), Sugihara and May, (1990), Sugihara et al. (1996), Fernandez-Rodriguez and Sosvilla-Rivero (1998), Gilbert and Bruneti (1997), and Cromwell et al. (2000). See also Barnett et al. (1997) for an excellent survey of nonlinear tests. Selected results confirm that the behavior of returns, particularly of futures prices, are highly complex. The estimated correlation dimension is high and little evidence exists of low-dimensional deterministic chaos (Decoster et al. (1992)). Such returns behavior is understandable, because noise and uncertainty play an important role in commodity price formation.

Important in dealing with risk is the formation of expectations. Although the rational expectations assumption is a standard feature of most work in modern macroeconomics and finance, more recent studies have found that the conclusions of this theory are often in conflict with empirical results. The latter suggest that to the contrary agents are heterogeneous and display bounded rationality. Examples include Frydman and Phelps (1983), Board (1994), Arthur (1994), Kyrtsou and Terraza (2001a,b), and Charella et al. (2000). Newly developed financial models thus attempt to show that the main cause of chaotic structures in asset prices is the heterogeneity of traders' expectations (see Brock and Hommes (1998), Lux (1995, 1998), and Gaunersdorfer (2000)). However, the discovery of chaotic processes is difficult; one can find randomness or noisy chaos, that is a dynamic chaos system disturbed by random noise. Modeling such processes also is difficult. Nonlinear models may see price fluctuations triggered by an interaction between a stabilizing force driving prices back towards fundamental values when the market is dominated by fundamentalists, while destabilizing forces drive prices away from their fundamental values when the market is dominated by speculative traders, i.e. when the long speculative to hedging ratio is several times unity. More recently Malliaris and Stein (1999) in examining futures price series provide insights as to how such problems can be dealt with; even if a system is totally (chaotic) deterministic, the resulting movements of price variability can be described statistically as random. This confirms the fact that chaotic series can mimic real financial series properties (see Kyrtsou and Terraza (2001c)).

In this study we attempt to examine this form of chaotic behavior for the case of metal futures prices. Metal trading activity of both producers and consumers involves substantial risks and further insights in to the formation of price expectations can be extremely helpful. This requires that we employ the BDS test (1987), the White (1989) neural network test along with the Lyapunov exponent (Wolf (1985), and Kantz (1994)), correlation-dimension methods (Grassberger and Procaccia (1983)), the modified rescaled range test by Lo (1991) together with the modified version by Moody and Wu (1996), and finally a spectral version of the fractional integration test (Geweke and Porter-Hudak, (1983)). Our results confirm that price behavior is noisy chaotic. This implies that corresponding models could be constructed such that prices could follow a mean process that is a dynamic chaotic with a variance that follows a GARCH process. As demonstrate by Kyrtsou and Terraza (2001b) in the case of stock prices, models of this type could be constructed to assist in forecasting prices over short run but not over long run time periods.

This paper consists of the following parts : Research methodology, Data and empirical results, and Conclusions.

2 Research Methodology

2.1 The BDS test

The BDS test provides a preliminary step to determine whether a time series process does or does not have observations that are independently and identically distributed (i.i.d). Brock, Dechert and Scheinkman (1987) have proposed a test of the i.i.d hypothesis based on the Grassberger and Procaccia correlation integral. This test compares the null hypothesis that a series is i.i.d against the alternative hypothesis that a series is linearly or non linearly correlated.

For a time series $\{X_t\}_{t=1,......T}$ featuring m-histories $X_t^m=(X_t, X_{t-1},...........X_{t+m-1})$, the correlation integral containing X is defined by:

$$C(\varepsilon,m,T_m) = \frac{1}{T_m(T_m-1)} \sum_{i,j=1}^{T_m} H(\|X_i^m - X_j^m\|) \qquad (1)$$

Here T_m = T-m+1 is the number of m-histories $X_t^m=(X_t, X_{t-1},...........X_{t+m-1})$ constructed from the sample of length T; H is the Heaviside function given by:

$$H(\|X_i - X_j\|) = \{1 \quad \text{if} \quad \|X_i - X_j\| < \varepsilon \text{ and } \quad 0 \quad \text{if} \quad \|X_i - X_j\| \geq \varepsilon\}.$$

Let X_t be an i.i.d series and suppose that $\sigma^2_m>0$; in this case

$$T_m^{1/2}\ [\ C(\varepsilon,m,T) - (C(\varepsilon,m,T))^m\ ] \xrightarrow{d} N(0,\ \sigma^2_m\ ) \text{ with } T_m \to \infty$$

where the expression "$\xrightarrow{d} N(0,\ \sigma^2_m\ )$" means: "convergence in distribution to $N(0,\sigma^2_m)$" and $N(0,\sigma^2_m)$ denotes the normal distribution with mean 0 and variance σ^2_m.

Considering that $C(\varepsilon,1\ ) \xrightarrow[T_m \to \infty]{} [C(\varepsilon,1\ )]^m$ [i.e. Denker et Keller (1986, theorem1 and (3,9))], equation (1) can be rewritten as:

$$W(\varepsilon,\ m) = T_m^{1/2}\ [C(\varepsilon,m) - (C(\varepsilon,1\ ))^m] / \sigma_m(\varepsilon) \tag{2}$$

Under the null hypothesis, X_t is i.i.d. and N(0,1). Note that $W(\varepsilon,m)$ is a function of two unknowns: the embedding dimension m, and the radius ε. There is an important relation between the choice of m and ε concerning the properties of a small sample for the BDS statistic. For a given m, ε cannot be too small, because in the opposite case there are insufficient pairs of points X_i, X_j, which would make the maximum of distance between them inferior or equal to ε (necessary condition for the calculation of the correlation integral). These small values of ε yield a slope systematically equal to m, because of the problem of noisy chaos [Brock and Dechert (1987)]. Inversely, ε must not be too large; otherwise the correlation integral contains too many observations.

Barnett and Choi (1989) suggest selecting a small value for ε, without allowing it to reach zero. This implementation of a lower limit guards against noise in the data. Hsieh (1989) defines ε in terms of multiples of the standard deviation of a given time series. These multiples are 0.50, 0.75, 1.00, 1.25, 1.50. The same multiples given by Girerd-Potin and Taramasco (1994) are 0.25, 0.50, 0.75, 1.00, 2.00, and 4.00. Brock, Hsieh and LeBaron (1992) instead use values of 0.25, 0.50, 1.00, 1.50, 2.00[1]. As Liu et al. (1992) indicate, the choice of ε is crucial, since different selected ranges of values of ε can lead to different conclusions. The authors suggest selecting m to belong to the interval [2,5].

Whatever the choice of ε and given the value of m, we calculate the W statistic. The obtained values of $|W|$ are to be compared with the theoretical value 1.96 of the normal distribution at the 5% level. If the estimated value is higher than 1.96, then the null hypothesis of independence in X is rejected. This rejection can result from:

1. either a non-stationary structure of the considered series, or
2. a structure of dependence resulting from a stochastic linear process (e.g. ARMA), or

[1] They have reached the best results when the ratio ε/σ varies between 0.50 and 2.

3. a structure of dependence issued from a nonlinear stochastic process (e.g. TAR, NMA, ARCH, GARCH, EGARCH), or
4. a structure of dependence issued from a nonlinear deterministic process (e.g. Hénon map, logistic equation, Mackey-Glass equation).

In order to use the BDS test as a test of nonlinearity, it is necessary at the start for the time series of interest to be stationary and to lack any linear structure. It is possible to eliminate any such linear dependence by filtering the data and by applying the BDS test to the residuals of an autoregressive model estimated from the initial stationary series.

2.2 The White neural network test

White (1989) has proposed that a time series can be fitted by a single hidden layer feedforward network, such as that described in Figure 1. In this network the input units ("sensors") send signals x_i, where i= 1,…….,k, along links ("connections") that amplify the original signals by the "weights" γ_{ji} (or "connection strengths"). The "hidden" (the intermediate) processing unit j "observes" signals $x_i\gamma_{ji}$, where i= 1,…….,k. Afterwards, the hidden units sum the arriving signals $\tilde{x}'\gamma_j$, with $\tilde{x}$ = (1, x_1,……….x_k)' and γ_j = (γ_{j0}, γ_{j1},………..,γ_{jk})'. The result is an output " activation " $\Psi(\tilde{x}'\gamma_j)$, where Ψ is a nonlinear "activation" or "squashing" function (usually the logistic distribution function $\Psi(\lambda)=(1+e^{-\lambda})$, $\lambda\in\Re$).

The network output can be defined as follows:

$$f(x,\delta)=\beta_0+\sum_{j=1}^{q}\beta_j\Psi\left(\tilde{x}'\gamma_j\right),\ q\in N. \tag{3}$$

where β_0,……, β_q are hidden to output weights ;Ψ ($\tilde{x}'\gamma_j$) are the hidden unit signals, j = 1,…….,q ; and δ = (β_0,….,β_q,…..,γ'_1,……,γ'_q)'. A single hidden-layer feedforward neural network can then be used to determine whether any nonlinear structure remains in the residuals of an autoregressive (AR) process fitted to a given time series This network output is given by:

$$o=\tilde{x}'\theta+\sum_{j=1}^{q}\beta_j\Psi\left(\tilde{x}'\gamma_j\right) \tag{4}$$

Finally the null hypothesis of interest specifies linearity in the mean relative to an information set[2]. The performance of the test depends on the following M statistic:

$$M_T = \left(\left(T^{-\frac{1}{2}} \sum_{t=1}^{T} \Phi_t \hat{e}_t \right) \hat{W}_T^{-1} \left(T^{-\frac{1}{2}} \sum_{t=1}^{T} \Phi_t \hat{e}_t \right) \right) \tag{5}$$

where $\hat{e}_t$ are the estimated residuals of the linear model, $\Phi_t = \left(\Psi\left(\tilde{x}_t'\Gamma_1\right), \ldots\ldots\ldots, \Psi\left(\tilde{x}_t'\Gamma_q\right)\right)'$; and Γ_i, i=1,…….,q, the hidden unit activations, are chosen at random. $\hat{W}_t$ is a consistent estimator of $W^* = \mathrm{var}\left(T^{-\frac{1}{2}} \sum_{t=1}^{T} \Phi_t e_t^* \right)$.

Implementing the test as a Lagrange multiplier test requires the following hypothesis formulation:

$$H_0: E(\Phi_t\, e_t^*) = 0 \quad \text{vs} \quad H_\alpha: E(\Phi_t\, e_t^*) \neq 0$$

For the case where M_T is asymptotically $\chi^2(q)$ under the null as $T \rightarrow \infty$, Bonferroni bounds provide an upper limit on the p-value. If $p_1, \ldots\ldots, p_k$ denote the ascending-ordered p-values corresponding to k draws from Γ, then the simple Bonferroni implies rejection of a linear null at the 100α% level if $p_i \leq \alpha/k$, so that, in the limit, the simple Bonferroni p-value is given by $\alpha = k\, p_i$. Hochberg (1988) suggests a modification to the Bonferroni method, which allows consideration of the p-values rather than just the largest, which may have led to a loss of power. The modified Hochberg-Bonferroni limit is given by $\alpha = \min_{i=1,\ldots,k}(m-i+1)p_i$, so that H_0 is rejected if there exists an i such as $p_i \leq \alpha/(m-i+1)$, i = 1,…,k.

[2] The process y_t is linear in mean conditional on X_t, if $P[E(y_t|X_t)=X_t'\theta^*] = 1$ for some $\theta^* \in R^k$. The alternative hypothesis is that y_t is not linear in mean conditional on X_t, if $P[E(y_t|X_t)=X_t'\theta]<1$ for all $\theta \in R^k$. When the alternative is true, then the process exhibits “neglected nonlinearity”. It is necessary to note that when a process is linear it is also linear in the mean, but the converse does not need to be true.

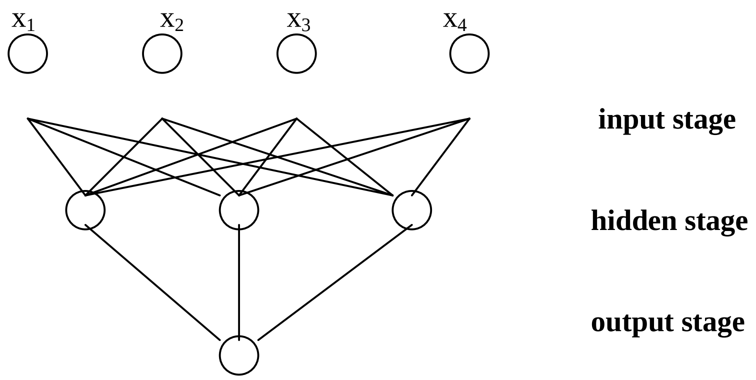

Figure 1.: Single hidden-layer feedforward network.

2.3 The correlation dimension test

The correlation dimension was introduced by Grassberger and Procaccia (1983). The correlation dimension is based on the idea that if an attractor is chaotic, then two points (X_i ,X_j) starting at different positions will be dynamically uncorrelated as a result of the property of sensitive dependence on initial conditions. However, since the points are on an attractor, they can approach each other but can never intersect. The correlation between points on an attractor can be defined in term of spatial correlation that is formally measured by the Euclidean distance.

Let $\{X_t\}$, t = 1,2.........,T be a sample from a strictly stationary process. The time series $\{X_t\}$ can be "embedded" in a m-space by constructing "m-histories". The correlation dimension can be calculated from the correlation integral given by:

$$C(\varepsilon,m,T_m) = \frac{1}{T_m\left(T_m - 1\right)} \sum_{i,j=1}^{T_m} H\left(\left\|X_i - X_j\right\|\right) \qquad i \neq j \tag{6}$$

as defined in (1).

The use of an Euclidean norm for computing the correlation dimension is considered not to be too restrictive. Brock (1986, theorem 2.4) has proved that the correlation dimension is independent of the choice of norm. Consider $C(\varepsilon,m,T_m)$ to be a double average of an indicator function. This integral can be thus computed as follows:

$$C(\varepsilon,m,T_m) = \frac{1}{T_m\left(T_m - 1\right)} \sum_{i=1}^{T_m}\sum_{j=1}^{T_m} H\left(\left\|X_i - X_j\right\|\right) \qquad i \neq j$$

Denker and Keller (1986) and Brock and Dechert (1987) show that:

$C(\varepsilon,m,T_m) \xrightarrow{d} C(\varepsilon,m)$ [i.e $C(\varepsilon,m,T_m)$ converges in distribution to $C(\varepsilon,m)$], when $T_m \to \infty$

Now let $\mathbf{x} \equiv (x_0, x_1, ..., x_{m-1})$, and $\mathbf{y} \equiv (y_0, y_1, ..., y_{m-1})$ and

$F_m(x_0, x_1, ..., x_{m-1}) \equiv \text{Prob}\{\alpha_t \le x_0, \alpha_{t+1} \le x_1, ..., \alpha_{t+m-1} \le x_{m-1}\} \equiv \text{Prob}\{\alpha_t^m \le \mathbf{x}\}$

$C(\varepsilon,m)$ can thus be defined by:

$$C(\varepsilon,m) = \int_{R^m} \int_{R^m} I(x, y; \varepsilon) dF_m(x) dF_m(y) = \int_{R^m} \left[F_m(x_0 + \varepsilon) - F_m(x_0 - \varepsilon)\right] dF_m(x) \qquad (7)$$

For the case of $C(\varepsilon,1)$, then (7) becomes

$$C(\varepsilon,1) = \int_R \int_R I(x_0, y_0; \varepsilon) dF_1(x_0) dF_1(y_0) = \int_R \left[F_1(x_0 + \varepsilon) - F_1(x_0 - \varepsilon)\right] dF_1(x_0).$$

For the case where $\{\alpha\}$ is i.i.d, then:

$$F(\mathbf{x}) = \prod_{i=0}^{m-1} (x_i). \qquad (8)$$

Finally Brock and Baek, (1991) combine the relations (7) and (8)such that:

$$C(\varepsilon,m) = [C(\varepsilon,1)]^m$$

Let the correlation integral measure the fraction of total number of pairs $(x_i, x_{i+1}, ..., x_{i+m-1})$, $(x_j, x_{j+1}, ..., x_{j+m-1})$, such that the distance between them is no more than ε. Then the correlation dimension is defined as:

$$d_c = \lim_{\varepsilon \to 0} \frac{\ln C(\varepsilon)}{\ln \varepsilon} \qquad (9)$$

For the small values of ε, Grassberger and Procaccia (1983) establish that the spatial correlation $C(\varepsilon,m)$ grows according to the power law:

If $d_m = \lim_{\varepsilon \to 0} \frac{\ln C(\varepsilon, m)}{\ln \varepsilon}$, then $\ln C(\varepsilon,m) \approx d_m \ln\varepsilon \Leftrightarrow \ln C(\varepsilon,m) \approx \ln \varepsilon^{d_m}$ $C(\varepsilon,m) \approx \varepsilon^{d_m}$, and $C(\varepsilon,m)$ grows exponentially.

One can now define what is known as the correlation dimension. It is necessary to notice that when the embedding dimension m increases, the dimension d_m is reached, such that d^*_c is the estimate of the true correlation:

$$d^*_c = \lim_{m\to\infty} d_m$$

The method of the correlation dimension represents a very important diagnostic procedure for distinguishing between determinism and stochasticity. If d_m tends to be a constant as m increases, then d_m yields an estimate of the correlation dimension of the attractor, namely d^*_c. In this case, the time series are consistent with deterministic behavior. If d_m increases without bound as m increases, this suggests that the underlying series are stochastic [3].

2.4 The Lyapunov exponent test

The method of the Lyapunov exponent can be employed to determine whether the process generating a time series is chaotic. The approach is based on the idea that the distance between two points is described by the largest Lyapunov exponent. The Lyapunov exponent measures the average rate of contraction (when negative) or expansion (when positive) on an entire attractor. The exponents can be positive or negative, but at least one exponent must be positive for an attractor to be classified as chaotic. If the distance grows exponentially, this is evidence of chaos since it shows that the process exhibits sensitive dependence to initial conditions. Thus where λ is the largest Lyapunov exponent, the criteria are:

$$\begin{aligned} &\text{stochasticity} \quad \text{if} \quad \lambda < 0, \\ &\text{chaos} \quad \text{if} \quad \lambda > 0 \end{aligned}$$

In the one-dimensional case, where $X_{t+1}=f(X_t)$ with $t \in N$, $X \in R^n$, the Lyapunov exponent λ can be defined (Lorenz, (1989)) by $\lambda^{(N)} = (1/N)\log_2(\Lambda^{(N)})$, where $\Lambda^{(N)}$ are the eigenvalues of the n-dimensional Jacobian matrix $J^{(N)}$. In general, all Lyapunov exponents can be calculated according to the following equation [see Wolf et al., (1985)]:

[3] Ruelle (1990) argues that a chaotic series can only be distinguished if it has a correlation dimension well below $2\log_{10}T$, where T is the size of the data set, suggesting that with economic time series the correlation dimension can only distinguish low dimensional chaos from high dimensional stochastic processes.

$$\lambda_{\mathrm{i}} = \lim_{N\to\infty} \frac{1}{N} \log_2(\Lambda_{\mathrm{i}}^{(\mathrm{N})}) \tag{10}$$

When applying this method to financial series, many authors have confirmed the difficulty of pollution from high frequency noise. The largest Lyapunov exponent λ_1 tends to be greater than the true exponent and its convergence to a value appears to be difficult or impossible.

Kantz (1994) has tried to solve this problem by constructing a new algorithm for the estimation of λ_1. Similar to Wolf et al. (1985), he makes use of the fact that the distance between two trajectories typically increases with a rate given by the maximal Lyapunov exponent. This divergence rate of trajectories naturally fluctuates along the trajectory, with the fluctuations given by the spectrum of effective Lyapunov exponents. The maximal exponent λ_τ is defined to be:

$$\lambda_\tau(\mathrm{t}) = \lim_{\varepsilon\to 0}\frac{1}{\tau}\ln\left(\frac{\left|\chi(\mathrm{t}+\tau)-\chi_\varepsilon(\mathrm{t}+\tau)\right|}{\varepsilon}\right) \tag{11}$$

where $\chi(t)$ is the time evolution of some initial condition $\chi(0)$ in an appropriate state space, t is time, and τ is relative time referring to the time index of the starting point , and $\varepsilon\left|\chi(0)-\chi_\varepsilon(0)\right|$. $\chi(t)-\chi_\varepsilon(t)=\varepsilon\omega_u(t)$, where $\omega_u(t)$ is the local eigenvector associated with the maximal Lyapunov exponent λ_{max}. By definition the average of $\lambda_\tau(t)$ along the trajectory is the true Lyapunov exponent.

The method of Kantz requires constructing the following equation to provide the curve $S(\tau)$. The maximal Lyapunov exponent is the slope of this curve in the scaling region.

$$S(\tau) = \frac{1}{T}\sum_{t=1}^{T}\ln\left(\frac{1}{|U_t|}\sum_{i\in U_t} dist(\chi_t,\chi_i;\tau)\right) \tag{12}$$

where: U_t is the neighborhood set and $dist(\chi_t,\chi_i;\tau)$ defines the distance between a reference trajectory χ_t and a neighbor χ_i after the relative time τ. The distance behaves such that dist = $\sum_i \alpha_i \exp(\lambda_i t)$, where λ_i are the effective Lyapunov exponents.

When noise is present in the data, the slope of the curve S(τ) changes as follows:

$$s(\tau) \approx \lambda + \left[\frac{\sigma_{i,\tau}}{dist(x_t, x_i; \tau)}\right]_t - \left[\frac{\sigma_{i,\tau-1}}{dist(x_t, x_i; \tau-1)}\right]_t$$

λ is the estimate of the maximal Lyapunov exponent and $\sigma_{i,\tau}$ is the standard deviation of the noise. S(τ) does not contain the embedding dimension explicitly, but nevertheless it enters. This requires that one fix a dimension m for the delay trajectories[4].

The Lyapunov estimator is the resulting value that converges on λ_1. If the convergence is not possible, then there are three explanations: (1) either the time series is linear, or (2) the sample is small, or (3) the noise level is very high.

2.5 The Lo rescaled range test

To detect long-range or "strong" dependence, Hurst (1951) has suggested using a test defined by the range and standard deviation (R/S statistic), also called the "rescaled range". The R/S statistic was later refined by Mandelbrot and Wallis (1969). $\overline{Q}_n$ represents the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. Consider a sample of returns x_1, x_2,……..,x_n and let $\overline{x}_n$ denote the sample mean (1/n) $\sum_j x_j$.

$$\overline{Q}_n = \frac{1}{s_n}\left[\underset{1\le k\le n}{Max}\sum_{j=1}^{k}\left(x_j - \overline{x}_n\right) - \underset{1\le k\le n}{Min}\sum_{j=1}^{k}\left(x_j - \overline{x}_n\right)\right] \tag{13}$$

where s_n is the usual standard deviation estimator:

$$s_n = \left[\frac{1}{n}\sum_j \left(x_j - \overline{x}_n\right)^2\right]^{1/2}$$

While studying long memory structures in stock price series using R/S analysis, Lo (1991) found that rejections of the null hypothesis (that the time series is a random walk versus the alternative that the process exhibits long memory) based on long time scales can be erroneous and can instead be due to bias induced by short term dependencies. To remove this mean bias, he proposed a modified R/$\tilde{S}$

statistic. His motivation was that if x_t is subject to short-term dependence, the autocovariances of x_t will not be equal to zero, and the range R cannot simply be normalized by the standard deviation alone. The rescaling term proposed by Lo includes weighting the covariances up to lag q and has the form:

$$\tilde{Q}_n = \frac{1}{\hat{\sigma}_n(q)}\left[\underset{1\le k\le n}{Max}\sum_{j=1}^{k}\left(x_j - \bar{x}_n\right) - \underset{1\le k\le n}{Min}\sum_{j=1}^{k}\left(x_j - \bar{x}_n\right)\right] \tag{14}$$

where $\hat{\sigma}_n^2(q) = \frac{1}{n}\sum_{j=1}^{n}\left(x_j - \bar{x}_n\right)^2 + \frac{2}{n}\sum_{j=1}^{q}\omega_j(q)\left\{\sum_{i=j+1}^{n}\left(x_i - \bar{x}_n\right)\left(x_{i-j} - \bar{x}_n\right)\right\} = \hat{\sigma}^2 + 2\sum_{j=1}^{q}\omega_j(q)\hat{\gamma}_j$

with $\omega_j(q) = 1 - \frac{j}{q+1}$; $q<n$; $\hat{\sigma}^2$ and $\hat{\gamma}_j$ are the usual sample variance and autocovariance estimators of x.

Since $\overline{Q}_n$ and $\tilde{Q}_n$ differ solely in how the range is normalized, the limiting behavior of Lo's modified R/S statistic and Hurst's original will only coincide when $\hat{\sigma}_n(q)$ and s_n are asymptotically equivalent. From the definitions of $\hat{\sigma}_n(q)$ and s_n, it is apparent that both will generally converge in probability to different limits in the presence of autocorrelation. Therefore, under the null hypothesis the statistic $\tilde{Q}_n/\sqrt{n}$ will converge to the range V of a Brownian bridge multiplied by some constant[5].

2.6 The Moody and Wu modified R/S test*

Moody and Wu (1996) have found that Lo's test statistic is itself biased and causes several problems concerning short time scales when attempting to correct the mean bias of the range R, including distortion of the Hurst exponent. While Lo's approach focuses on the actual value of the R/$\tilde{S}$ statistic for a given time scale n, Hurst and Mandelbrot test for long term dependency by comparing the slope of the R/S curve to 0.5 (i.e. a random walk process). Moody and Wu's empirical results show, however, that biases in the definitions of R, S and $\tilde{S}$ can lead to errors in the estimates of R/S, R/$\tilde{S}$, and H for short time scales n or when short dependencies are present in the series. These errors can lead to misleading and sometimes inconsistent results.

In order to address these problems, Moody and Wu have proposed an unbiased rescaling factor S^* that corrects for mean bias in the range R due to short-term dependencies without inducing the

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[4] For more details in the choice of embedding dimension, see Kantz (1994).
[5] The table of V's distribution (F_V) is given by Lo (1991).

distortions on short time scales that S and Lo's $\tilde{S}$ do. In that case the standard unbiased estimate of the variance is given by:

$$\hat{\sigma}_n^2 = \frac{1}{n-1}\sum_{j=1}^{n}\left(x_j - \bar{x}_n\right)^2 \qquad (15)$$

The new unbiased rescaling factor with weighted covariances up to lag q is:

$$S^* = \left\{\left[1 + 2\sum_{j=1}^{q}\omega_j(q)\frac{n-j}{n^2}\right]\hat{\sigma}_n^2 + \frac{2}{n}\sum_{j=1}^{q}\omega_j(q)\sum_{i=j+1}^{n}\left(x_i - \bar{x}_n\right)\left(x_{i-j} - \bar{x}_n\right)\right\}^{1/2} \qquad (16)$$

Here $\omega_j(q)$ is the weighting function as defined by Lo (1991) and q<n. When q = 0, S^* reduces to the unbiased standard deviation $\hat{\sigma}$ and not to the Mandelbrot's R/S analysis, as is the case for the Lo's R/$\tilde{S}$ statistic[6].

2.7 The fractional integration test

A time series **x**={ $x_1,\ldots, x_T$} with mean μ follows an autoregressive fractionally integrated moving average process of order (p,d,q), denoted by ARFIMA(p,d,q), if:

$$\Phi(B)(1-B)^d(x_t-\mu)=\theta(B)\varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d}(0,\sigma^2_\varepsilon) \qquad (17)$$

where B is the backward-shift operator. Then $\Phi(B)=1-\phi_1B-\ldots-\phi_pB^p$, and $(1-B)^d$ is the fractional differencing operator defined by:

$$(1-B)^d = \sum_{k=0}^{\infty}\frac{\Gamma(k-d)B^k}{\Gamma(-d)\Gamma(k+1)} \qquad (18)$$

with Γ(.) denoting the gamma function. The parameter d is allowed to assume any real value.

The stochastic process **x** is both stationary and invertible, if all roots of Φ(B) and θ(B) lie outside the unit circle and $|d|<0.5$, since it possesses infinite variance (Granger and Joyeux (1980)). Assuming that -1/2<d<1/2 and d≠0, Hosking (1981) showed that the correlation function of an

[6] The empirical results in this paper, suggest that occasionally for q=0, the Lo and Moody and Wu statistics give identical values.

ARFIMA process is proportional to j^{2d-1} as $j \to \infty$. Consequently, the autocorrelations of the ARFIMA process decay hyperbolically to zero as $j \to \infty$ which is contrary to the faster, geometric decay of a stationary ARMA process. For $0<d<1/2$, the ARFIMA process is said to exhibit long memory. The process exhibits short memory for $d = 0$ and intermediate memory for $d<0$. The existence of a fractional order of integration can be determined by testing for the statistical significance of the sample differencing parameter d. To estimate d and to perform hypothesis testing, we employ the spectral regression method suggested by Geweke and Porter-Hudak (1983).

3. Data and Empirical Results

The metal futures prices we study are derived from London Metal Exchange trading for aluminium, nickel, tin, zinc, and lead. They represent near three-month contracts and consist of daily closing values. All series begin on January 1989 and end on April 1998 (n = 2355), except for the tin series that begin on August 1989 (n = 2209). Starting with the Dickey-Fuller test, we found that all series display nonstationarity in price levels. To ensure that the data are stationary, first differences of the log of each price series were employed, i.e. returns in the form of $X_t = \Delta \log P$. The returns series exhibit dependence in the fourth cumulants (kurtosis) and third cumulants (skewness), and they show strong signs of non-normality[7].

The results of applying the BDS test to the returns series are given in Table 1. It must be noted that for simplicity reasons we report the results of the W statistic only for $\varepsilon=0.25\sigma$. Looking at the absolute values, we can easily conclude that the hypothesis of independence is strongly rejected (the estimated values are clearly greater than the critical value of 1.96). After filtering the series for linear dependence, we applied the BDS test to the residuals and reject the null hypothesis of a possible i.i.d process[8]. This rejection, since it does not derive from a structure of linear dependence, leads to the conclusion that the processes underlying the five price series are nonlinear. The same results are obtained from applying the White neural network test. From Table 2 we also reject the null hypothesis for linearity in the mean, even though the M statistic or the Hochberg-Bonferroni limit is reached .

These nonlinear processes can be of a stochastic or of a deterministic nature. That is, the shocks that perturb the market can be respectively exogenous or endogenous. According to each approach, there are as many possibilities for forecasting the returns series in the long or short term as there are different forecasting methods. In order to distinguish between stochasticity or determinism, we have applied the correlation-dimension test along with the method of Lyapunov exponents. The noise problem that tends to "destroy" the robustness of Lyapunov exponents has been resolved by using a

[7] The descriptive statistics of the returns series are available upon request from the authors.
[8] The results of BDS test as test for nonlinearity are available upon request, like in the previous case.

new algorithm, proposed by Kantz (1994). Nevertheless, Wolf's (1985) algorithm is also applied so that we can make comparisons between an estimated exponent with a large noise level and one with a small noise level.

The results of estimating the correlation dimension and the two Lyapunov exponent algorithms are presented in Tables 3, 4, 5. The correlation dimension (CD) for the aluminium, nickel and zinc returns is infinite, while for the tin and lead returns, the CD is approximately equal to 5.958 and 6.097, respectively. Consequently we can characterize this behavior as stochastic. To further interpret the underlying generating process, we recompute the relevant Lyapunov exponents. From Table 4 employing Wolf's algorithm, we see that the Lyapunov exponents for all series converge to a small but positive value. Taking only this result into account, one would normally consider that the underlying process is deterministic chaotic. This conclusion however, can be incorrect because of noise effects. Table 5 now employing Kantz's algorithm reports new values for the Lyapunov exponents. In this case, the aluminium, nickel and tin returns seem to converge to a slightly negative value, while the zinc and lead series converge clearly to a small but positive value. Recall that Gençay and Liu (1996) have obtained negative Lyapunov exponents for three noisy chaotic models (noisy logistic map, noisy Hénon map, and noisy Mackey-Glass delay equation.[9] The fact that the Lyapunov exponents are positive for the zinc and lead returns might be due to the existence of an underlying high dimensional chaotic system, one which can generate similar random behavior because of hidden dimensionality. These various results when taken together would suggest that the processes underlying the metal price series are not purely stochastic.

Our next step is to evaluate the memory in the returns series by applying the modified R/S statistic by Lo as well as by Moody and Wu and the fractional integration test. As shown in Table 6 the lack of statistical significance for the Lo and the Moody and Wu statistics indicate that the behavior of the metals price returns is consistent with the null hypothesis of short memory. Nevertheless, we would like to emphasize that these tests cannot always detect chaotic structures in time series. For example, Lo (1991) shows that for a Mackey-Glass (1977) process, the value of the V statistic being inferior to the critical value, i.e. the process exhibits short memory. Therefore, we know that a chaotic process can exhibit long memory due to the property of its sensitivity to initial conditions.

Looking at Table 7, we can easily conclude from the t-student test (comparing t-estimates to a critical value of 1.96), that for aluminium, nickel and lead $d=0$, and for tin and zinc $d<0$. That is, the aluminium nickel and lead returns series exhibit short memory, while the tin and zinc returns reflect intermediate memory or anti-persistent processes. Remember that when $d=0$, short memory describes the low correlation structure of a series, since the correlations among the returns at long lags become negligible. An anti-persistent process, however, constitutes an intermediate case between short and

[9] A noisy chaotic system is a chaotic system having a high dimension.

long memory process. Nonetheless, we must be cautious, because chaotic processes can also have a value of d inferior to zero (the case for the logistic equation with μ=4) as well as d equal to zero (the case for the Lorenz equation).

4. Conclusions

In this study, our efforts have been directed to providing an interpretation of risk and price expectations by focusing on the discovery of noisy chaotic price generating processes, not previously detected in metals futures prices. The results obtained from applying the above tests to the returns series would suggest the presence of nonlinear stochastic processes with short memory behavior for the aluminium, nickel and lead price returns series, implying that they are predictably mean-reverting. In contrast, the results from the tin and zinc returns series suggest anti-persistent processes. A nonlinear stochastic short memory model, such as that of an ARCH process, would explain the observed nonnormality, nonperiodic cycles, and spikes in the price series to result from the dependence found in the variances, or risk of the series. These variances describe the risk surrounding a price series, that is typically encountered in commodity trading. The interactions of various risk elements in the history of returns lead to a path that also contributes to the irregular behavior of that returns series.

Our findings of short and intermediate memory should thus enable traders to better interpret the underlying processes of the aluminium, copper, nickel, tin and zinc returns series to be primarily noisy (or stochastic) chaotic. This means that returns series can be modeled by a chaotic process buffeted with dynamic noise (Kyrtsou and Terraza (2001a,b)). The root cause of chaotic dynamics in commodity markets is expectations heterogeneity (Nishimura, (1998)) or interactions between heterogeneous traders (Frankel and Froot (1988), De Long et al. (1990), Wang (1993, 1994), and Dacorogna et al. (1995) . As Nishimura (1998) has shown, the heterogeneity of agents' expectations makes prices excessively sensitive (or more volatile) in homogenous commodity markets with non-trivial production. In this way, price volatility can be interpreted endogenously. In contrast to the findings of Cromwell et al. (2000) and Gilbert and Bruneti. (1997), we not only consider volatility dynamics, but we also study a "mixed" nonlinearity, i.e. in the mean and the conditional variance. Thus, as Kyrtsou and Terraza (2001b) have shown, when we take into account the hidden nonlinear patterns in the error term and the deterministic component jointly, we can obtain better forecasts for price series.

When the behavior of a returns series is found to be chaotic, it is possible to make predictions in the short run but not in the long run. This suggests the potential of applying the Mackey-Glass model to the forecasting of commodity futures price series, a task waiting to be accomplished. For traders who have higher risk bearing capabilities, this suggests that optimal locations on their mean-variance price efficiency frontier are more likely to be found in a market that has a chaotic structure than in one displaying pure randomness.

Tables

Table 1: BDS test results

	Aluminium	Nickel	Tin	Zinc	Lead
m=2	6.71	10.52	13.43	5.06	15.76
m=3	5.58	15.59	13.04	6.85	14.09
m=4	-21.43	20.78	11.99	6.81	12.64
m=5	-13.85	27.49	11.51	7.47	11.72

Table 2: White test results[1]

Series	M_t (2)	H-B Limit (3)
Aluminium	16.88	0.00075
Nickel	35.99	0
Tin	334.31	0
Zinc	8.82	0.02431
Lead	8.07	0.04524

1: We present the results obtained, using 3 principal components. We also note that the test for linearity is not against general nonlinearity of the process but against nonlinearity in the mean.
2: The critical value of the test at the 5% level is 5.99. If M_t is greater than the critical value we reject the null hypothesis.
3: Hochberg's modification is used here, defined by the rule "reject H_0 at the α level if there exists an i such as $P_i \leq \alpha/(m-i+1)$, i = 1,……,k".

Table 3: Correlation-dimension method results

	Aluminium	Nickel	Tin	Zinc	Lead
m=2	2.062	2.050	2.083	2.024	2.090
m=3	2.994	2.989	3.010	2.952	2.982
m=4	3.849	3.691	3.858	3.694	3.791
m=5	4.550	4.329	4.254	4.428	4.512
m=6	5.043	4.784	4.889	4.625	5.008
m=7	5.559	5.264	5.413	4.979	5.540
m=8	5.803	5.491	5.625	5.153	5.701
m=9	6.202	5.727	5.958	5.392	6.097
m=10	6.540	5.956	5.776	5.643	6.042

Table 4: Lyapunov exponent method results, using the Wolf's algorithm (1985)

	Aluminium	**Nickel**	**Tin**	**Zinc**	**Lead**
m=2	0.844	0.847	0.157	0.053	0.005
m=3	0.628	0.602	0.097	0.107	0.037
m=4	0.338	0.312	0.110	0.064	0.014
m=5	0.213	0.220	0.115	0.067	0.007
m=6	0.162	0.139	0.101	0.032	0.016
m=7	0.110	0.117	0.087	0.035	0.007
m=8	0.087	0.088	0.120	0.027	0.038
m=9	0.069	0.080	0.095	0.028	0.003
m=10	0.067	0.072	0.074	0.026	0.014

Table 5: Lyapunov exponent method results, using the Kantz's algorithm (1994)

	Aluminium	**Nickel**	**Tin**	**Zinc**	**Lead**
m=2	-0.002474	-0.001025	-0.05268	0.000076	0.008431
m=3	0.000014	-0.008765	0.006163	0.171679	0.000799
m=4	0.000634	-0.035707	0.003953	0.001544	0.007276
m=5	0.001002	-0.001107	0.004447	0.205897	0.008379
m=6	-0.003955	-0.060117	0.009952	0.218431	0.003427
m=7	-0.001233	-0.005145	0.012629	0.221021	0.005864
m=8	0.001079	-0.001586	0.007915	0.237138	0.008334
m=9	0.000269	0.000642	-0.051148	0.238011	0.009057
m=10	-0.002139	-0.001297	-0.08936	0.239196	0.008918

Table 6: R/S test results for the metal returns series

	Lags for q			
Methods	**q=0**	**q=2**	**q=4**	**q=6**
Aluminium				
Lo	1.9828	1.253	1.2421	1.20
Moody&Wu	1.97	1.29	1.2407	1.204
Nickel				
Lo	1.42	0.9306	0.888	0.8659
Moody&Wu	1.42	0.9287	0.889	0.8657
Tin				
Lo	1.3303	0.8915	0.8464	0.8301
Moody&Wu	1.332	0.8887	0.8442	0.827
Zinc				
Lo	1.3367	0.8398	0.8123	0.7983
Moody&Wu	1.3367	0.8496	0.8105	0.7958
Lead				
Lo	1.4727	0.9356	0.8933	0.8789
Moody&Wu	1.4727	0.9345	0.8913	0.8706

The critical value for the V statistic is 1.75 at the 5% level and 1.62 at the 10% level

Table 7: Fractional Integration test results
(Geweke and Porter-Hudak, 1983)

Series	**d for $T^{0.45}$**	**d for $T^{0.5}$**	**d for $T^{0.55}$**
Aluminium	0.2817 (2.0932)	0.0607 (0.5753)	0.0580 (0.6885)
Nickel	0.0235 (0.1745)	-0.0905 (-0.8578)	0.0343 (0.4080)
Tin	-0.3022 (-2.2025)	-0.2675 (-2.4720)	-0.3009 (-3.5173)
Zinc	-0.2215 (-1.6459)	-0.2354 (-2.2305)	-0.3743 (-4.4462)
Lead	0.0214 (0.1584)	0.0816 (0.7704)	0.2410 (2.8391)

For $d \in (0, 0.5)$ the series is a long memory or persistent process. For $d \in (-0.5, 0)$ the series is an antipersistent or intermediate memory process, while if d = 0 we have a short memory process. In order to test if d = 0 or $d \neq 0$ we construct a t-student test. The t-statistic values are given below the values of d, constructed imposing the known theoretical error variance of $\pi^2/6$. If the obtained value is less than 1,96 then we accept the null hypothesis d = 0.

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