splm

spatial panel data models in R

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Preliminary "assumptions"

The present "workshop" assumes that you are familiar with:

- Panel data models
- Cross-sectional spatial models. In particular:
  - Estimation theory for the spatial lag model
  - Estimation theory for the spatial error model
- Basic R knowledge (i.e. how to perform OLS using the lm function)

Outline

1. Motivation
2. ML estimators
   - Fixed effects models
   - Random effects models
3. GM estimator for spatial random effects models
   - Spatial panel data model
   - Spatial simultaneous equations model
4. Conclusion

Outline

Growing theory on Spatial Panel Data

There are many reasons to develop an R library for spatial panel data:

- Spatial panel data are probably one of the most promising but at the same time underdeveloped topic in spatial econometrics.
- Recently, a series of theoretical papers have appeared developing estimation procedure for different models.
  [Kapoor et al., 2007, Anselin et al., 2008, Elhorst, 2003, Yu et al., 2008, Yu and Lee, 2007, Pesaran and Tosetti, 2007, Lee and Yu, 2009, Lee and Yu, 2008, Lee and Yu, 2007, Baltagi et al., 2007a, Elhorst et al., 2009, Elhorst, 2005]
- Theoretical papers developing specification tests
  [Baltagi et al., 2007b, Baltagi et al., 2003, Baltagi et al., 2009, Baltagi and Liu, 2008]
- Although there exists libraries in R, Matlab, Stata, Phyton to estimate cross-sectional models, to the best of our knowledge, there is no software available to estimate spatial panel data models (except for Elhorst' s codes in Matlab and Stata code on Prucha's web site)

why R?

- Clearly because we are fan....
- Because of the existence of two libraries: `spdep` and `plm`
  - `spdep`: extremely well known library for spatial analysis that contains estimation procedures for spatial cross sectional models
  - `plm`: recently developed panel data library to perform estimation of fixed and random effects models (and a number of other functionalities)

How do we take advantage of these existing libraries?

- From `spdep`: We are borrowing many ideas, such as:
  - `listw` type of objects
  - the general ML estimation strategy
- From `plm`, among other things:
  - how to handle the double (space and time) dimension of the data
  - `print` and `summary` methods

Next we will see the results...

Let's get started...

How to install the library?

The package is available from R-forge.
R-forge offers a central platform for the development of R packages, R-related software and further projects.

```
> install.packages("splm", repos = "http://R-Forge.R-project.org")

The downloaded packages are in
        /var/folders/TT/TTVayxI+EoGhVAbua1cBFE+++TI/-Tmp-//RtmpSx2YOc/downl
```

Load required library, Data, spatial weights matrix and formula

```
> library(splm)
```

All functionalities are illustrated by application to [Munnel, 1990] data on 48 US States observed over 17 years (we restrict the time range to observations between 1970 and 1974):

```
> data(Produc, package = "Ecdat")
> Produc <- Produc[Produc$year %in% 1970:1974, ]
```

The spatial weights matrix (binary contiguity) is available with the library

```
> data(usaww)
```

Munnell's data

The model formula is defined once for all and includes a constant term:

```
> fm <- log(gsp) ~ log(pcap) + log(pc) + log(emp) + unemp
```

[Munnel, 1990] estimates a Cobb-Douglas production function investigating the productivity of public capital in private production:

- gsp: gross state product
- pcap: public capital
- pc: private capital stock
- emp: capital input measured in terms of employment
- unemp: state unemployment rate measured to capture business cycle effects

Estimating the model by OLS, Munnell finds a productive and significant effects of public capital. [Baltagi and Pinnoi, 1995] prove that after controlling for state specific effects public capital has no role.

Outline

Fixed effects models regional effects

The traditional fixed effects model can be extended to include a spatially lagged dependent variable among the regressors as well as a spatially autocorrelated error term. The simplest representation of a pooled linear regression model with spatial fixed effects takes the following form:

$$y_{it} = \alpha_i + x_{it}\beta + \varepsilon_{it} \tag{1}$$

where $i = 1, \ldots, N$ refers to individuals, $t = 1, \ldots, T$ is the time index, and α_i are the individual specific effects for which it holds that $\sum_i \alpha_i = 0$ to be separately identifiable from the constant term.

FE spatial lag model

A fixed effect spatial lag model can be written in stacked form as:

$$y = \rho(I_T \otimes W_N)y + (\iota_T \otimes \alpha) + X\beta + \varepsilon \tag{2}$$

where ρ is a spatial autoregressive coefficient, W a non-stochastic spatial weight matrix and $\varepsilon_i \sim N(0, \sigma^2)$.

The model can be specified with an intercept, with time period fixed effects, with spatial fixed effects and with both time period and spatial fixed effects.

The general estimation procedure is based on maximum likelihood and clearly resembles the estimation theory for the cross sectional model (e.g. use of concentrated likelihood, auxiliary regressions).

The argument `effects`

The argument `effects` accepts four different options:

- `pooled`: only the (optional) constant term is included
- `spfe`: cross-sectional specific effects are included
- `tpfe`: time period specific effects are included
- `sptpfe`: both spatial and time period fixed effects are included

```
> res <- spfeml(fm, data = Produc, listw = mat2listw(usaww), effects = "spfe",
+     method = "eigen")
```

```
 Warning: x may not contain an intercept if fixed effects are specified
```

Results FE

This is how the results should look like...

```
> summary(res)

Spatial panel fixed effects lag model

Call:
spfeml(formula = fm, data = Produc, listw = mat2listw(usaww),
    effects = "spfe", method = "eigen")

Residuals:
    Min.  1st Qu.   Median  3rd Qu.     Max.
-0.08690 -0.00965  0.00200  0.01190  0.08790

Coefficients:
            Estimate Std. Error t-value  Pr(>|t|)
rho        0.2829236  0.0608287  4.6512 3.301e-06 ***
log(pcap)  0.0086765  0.0741993  0.1169  0.906912
log(pc)    0.2058315  0.0826620  2.4900  0.012773 *
log(emp)   0.5638186  0.0568597  9.9160 < 2.2e-16 ***
unemp     -0.0065048  0.0024507 -2.6542  0.007949 **
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1
```

The argument `method`

The argument `method` accepts three different options:

- `eigen` the Jacobian is computed using the approximation suggested in [Ord, 1975].
- `spam` use code from the `spam` library to calculate the determinant
- `Matrix` use code from the `Matrix` library to calculate the determinant

The last two options should be used only with large datasets. Statistical inference of the spatial parameter is provided only when method is `eigen`.

methods for extracting and printing fixed effects

The function *effects* extracts fixed effects from the `splm` object.
A `print` method is available for printing fixed effects:

```
> eff <- effects(res)
> print(eff)
Intercept:
            Estimate Std. Error t-value Pr(>|t|)
(Intercept)  1.41326    0.27947   5.057 4.26e-07 ***


Spatial fixed effects:
    Estimate Std. Error t-value Pr(>|t|)
1   0.560120   0.283370  1.9766 0.048083 *
2  -0.041968   0.278085 -0.1509 0.880041
3  -0.398785   0.269354 -1.4805 0.138734
4   0.812791   0.327866  2.4790 0.013174 *
5   0.155323   0.274431  0.5660 0.571406
6  -0.036531   0.275791 -0.1325 0.894622
7  -0.588049   0.252238 -2.3313 0.019736 *
8   0.084812   0.292903  0.2896 0.772156
9   0.548298   0.283311  1.9353 0.052951 .
10 -0.138523   0.255737 -0.5417 0.588051
11  0.296824   0.307571  0.9651 0.334515
12 -0.237844   0.291055 -0.8172 0.413825
```

FE lag model 2

A model with both spatial and time period fixed effects can be estimated simply by changing the argument `effects` to '`sptpfe`':

```
> res <- spfeml(fm, data = Produc, listw = mat2listw(usaww), effects = "sptpfe",
+     method = "eigen")
```

```
 Warning: x may not contain an intercept if fixed effects are specified
```

FE lag model 2

The output should look like this...

```
> summary(res)

Spatial panel fixed effects lag model

Call:
spfeml(formula = fm, data = Produc, listw = mat2listw(usaww),
    effects = "sptpfe", method = "eigen")

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-8.49e-02 -7.96e-03  6.73e-05  8.83e-03  7.41e-02

Coefficients:
            Estimate Std. Error t-value  Pr(>|t|)
rho        0.1272702  0.0644383  1.9751   0.04826 *
log(pcap) -0.1362886  0.0673498 -2.0236   0.04301 *
log(pc)    0.7606320  0.1023941  7.4285 1.099e-13 ***
log(emp)   0.7125036  0.0503681 14.1459 < 2.2e-16 ***
unemp     -0.0043368  0.0023693 -1.8304   0.06719 .
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1
```

`write` FE to disk

Finally, there is a method available to write the table of fixed effects to a file:

```
> write.effects.splm(eff, "effects")
```

The first argument is the object to write, the second argument is the name of the file.

FE error model

A fixed effects spatial error model can be written as:

$$\begin{aligned} y &= (\iota_T \otimes \alpha) + X\beta + u \\ u &= \lambda(I_T \otimes W_N)u + \varepsilon \end{aligned} \tag{3}$$

where λ is a spatial autocorrelation coefficient and ε is a well-behaved error term.
Again estimation is based on maximum likelihood and resembles the cross-sectional case (iterative procedure alternating concentrated likelihood and feasible GLS).

FE Error model

The syntax of the corresponding function is very similar to the spatial lag case:

```
> err <- spfeml(fm, data = Produc, listw = mat2listw(usaww), model = "error",
+     effects = "tpfe")
```

```
Warning: x may not contain an intercept if fixed effects are specified
```

FE Error model

the summary method produces...

```
> summary(err)

Spatial panel fixed effects error model

Call:
spfeml(formula = fm, data = Produc, listw = mat2listw(usaww),
    model = "error", effects = "tpfe")

Residuals:
   Min. 1st Qu.  Median 3rd Qu.    Max.
-0.2140 -0.0645 -0.0107  0.0622  0.2750

Coefficients:
           Estimate Std. Error t-value  Pr(>|t|)
lambda    0.5610811  0.0616840  9.0961 < 2.2e-16 ***
log(pcap) 0.2050535  0.0323292  6.3427 2.258e-10 ***
log(pc)   0.4168832  0.0215229 19.3693 < 2.2e-16 ***
log(emp)  0.4551632  0.0273435 16.6461 < 2.2e-16 ***
unemp     0.0028102  0.0048420  0.5804    0.5617
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1
```

`print` the effects

As before, time period fixed effects and the intercept can be recovered as follows:

```
> eff <- effects(err)
> print(eff)

Intercept:
             Estimate Std. Error t-value  Pr(>|t|)
(Intercept)  0.94734    0.10022  9.4526 < 2.2e-16 ***


Time period fixed effects:
    Estimate Std. Error t-value Pr(>|t|)
1 -0.0042712  0.1002616 -0.0426   0.9660
2 -0.0095216  0.1012519 -0.0940   0.9251
3  0.0074007  0.1009238  0.0733   0.9415
4  0.0240944  0.1005069  0.2397   0.8105
5 -0.0177023  0.1010374 -0.1752   0.8609
```

General Model (Baltagi et al., 2007)

Consider the following panel data regression model:

$$y_{it} = X'_{it}\beta + u_{it} \tag{4}$$

where

$$u_t = \mu + \varepsilon_t \tag{5}$$

and

$$\varepsilon_t = \lambda W \varepsilon_t + \nu_t \qquad \text{where} \qquad \nu_t = \rho \nu_{t-1} + e_t. \tag{6}$$

The disturbance term can also be rewritten, in matrix notation, as

$$u = (\iota_T \otimes I_N)\mu + (I_T \otimes B^{-1})\nu \tag{7}$$

where $B = I_N - \lambda W$. Henceforth: $\phi = \frac{\sigma^2_\mu}{\sigma^2_\varepsilon}$
The model considers serial correlation on each spatial unit over time, and spatial dependence between spatial units at each time period.

Restrictions

Depending on the restrictions on the vector of parameters one can differently combine error features giving rise to various nested specifications:

Table: Different model specifications that can be generated as special cases of the general specification.

	$\lambda \neq 0$ $\rho \neq 0$	$\lambda \neq 0$ $\rho = 0$	$\lambda = 0$ $\rho \neq 0$	$\lambda = 0$ $\rho = 0$
$\sigma^2_\mu \neq 0$	SEMSRRE	SEMRE	SSRRE	RE
$\sigma^2_\mu = 0$	SEMSR	SEM	SSR	OLS

Estimation

We implement an iterative procedure to obtain the maximum likelihood estimates. Starting with initial values for λ, ρ and ϕ, we obtain estimates for β and σ_e^2 from the first order conditions:

$$\hat{\beta} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}y \tag{8}$$

$$\hat{\sigma}_e^2 = (y - X\beta)'\Sigma^{-1}(y - X\beta)/NT. \tag{9}$$

The likelihood can be concentrated and maximized with respect to λ, ρ and ϕ. Then we use these estimates to update the expression for Σ^{-1} and repeat this process until convergence.

Computational issues

There are some computational issues related to this approach:

- We include the GLS within the objective function to be maximized (i.e. the function to be used as argument of the optimizer)
- The choice of the optimizer: `nlminb`
- Standard errors for β from GLS, numerical Hessian to perform statistical inference on the error components
- Initial values of the parameters (discussed next)

Initial values

We give three options to set the initial values of the parameters managed through the argument `initval`.

- The default is to start from a predefined vector of zeros.
- As an alternative the user can specify a vector of length equal to the number of parameters to be estimated
- If `initval` is set to `estimate` the initial value for each parameter will be retrieved from the estimation of one of the nested specifications

The argument `errors`

The arguments `errors` allows the estimation of the models presented in the previous table:

- `semsrre` the full model
- `semsr` spatial and serial correlation
- `srre` serial correlation and random effects
- `semre` spatial correlation and random effects
- `re` random effects
- `sr` serial correlation
- `sem` spatial error model

RE model - ML Estimation

To estimate the most general model the instruction would be:

```
> semsrremod <- spreml(fm, data = Produc, w = usaww, errors = "semsrre")
```

RE model - ML Estimation

and the results are...

```
> summary(semsrremod)

Spatial panel random effects ML model

Call:
spreml(formula = fm, data = Produc, w = usaww, errors = "semsrre")

Residuals:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.2260 -0.0554  0.0144  0.0177  0.0861  0.3420

Error variance parameters:
        Estimate Std. Error t-value Pr(>|t|)
phi    11.866193   7.384789  1.6068   0.1081
rho     0.946385   0.032514 29.1067   <2e-16 ***
lambda  0.725737   0.050658 14.3263   <2e-16 ***

Coefficients:
              Estimate Std. Error t-value  Pr(>|t|)
(Intercept)  1.4835380  0.2308139  6.4274 1.298e-10 ***
log(pcap)    0.0218138  0.0565093  0.3860   0.69948
log(pc)      0.4271098  0.0474670  8.9980 < 2.2e-16 ***
log(emp)     0.6215272  0.0473262 13.1328 < 2.2e-16 ***
unemp       -0.0056863  0.0023635 -2.4058   0.01614 *
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1
```

RE model - ML Estimation

..and for the spatial error model with random effects:

```
> semremod <- spreml(fm, data = Produc, w = usaww, errors = "semre")
```

RE model - ML Estimation

and the results...

```
> summary(semremod)

Spatial panel random effects ML model

Call:
spreml(formula = fm, data = Produc, w = usaww, errors = "semre")

Residuals:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-0.246000 -0.073700 -0.004910  0.000124  0.068900  0.345000

Error variance parameters:
        Estimate Std. Error t-value  Pr(>|t|)
phi    26.101512   6.550819  3.9845 6.763e-05 ***
lambda  0.596499   0.066678  8.9460 < 2.2e-16 ***

Coefficients:
              Estimate Std. Error t-value  Pr(>|t|)
(Intercept)  1.6555503  0.2133124  7.7612 8.416e-15 ***
log(pcap)    0.0264020  0.0514811  0.5128   0.60806
log(pc)      0.3962823  0.0436601  9.0765 < 2.2e-16 ***
log(emp)     0.6397099  0.0394073 16.2333 < 2.2e-16 ***
unemp       -0.0062619  0.0026267 -2.3839   0.01713 *
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1
```

RE model - ML Estimation

..and, finally, a model with spatial and serial correlation:

```
> semsrmod <- spreml(fm, data = Produc, w = usaww, errors = "semsr")
```

RE model - ML Estimation

and the results...

```
> summary(semsrmod)

Spatial panel random effects ML model

Call:
spreml(formula = fm, data = Produc, w = usaww, errors = "semsr")

Residuals:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.2230 -0.0574  0.0104  0.0157  0.0842  0.3210

Error variance parameters:
        Estimate Std. Error t-value  Pr(>|t|)
rho    0.9767801  0.0057731 169.195 < 2.2e-16 ***
lambda 0.6996718  0.0499021  14.021 < 2.2e-16 ***

Coefficients:
              Estimate Std. Error t-value  Pr(>|t|)
(Intercept)  1.4038291  0.2324995  6.0380 1.560e-09 ***
log(pcap)    0.0304352  0.0558261  0.5452  0.585630
log(pc)      0.4394936  0.0477604  9.2021 < 2.2e-16 ***
log(emp)     0.6030684  0.0478206 12.6111 < 2.2e-16 ***
unemp       -0.0062612  0.0023664 -2.6458  0.008149 **
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1
```

Outline

Kapoor et al. (2007)

The model can be written as:

$$y_N = X_N\beta + u_N \tag{10}$$

where the disturbance term follows a first order spatial autoregressive process of the form:

$$u_N = \rho(I_T \otimes W)u_N + \varepsilon_N \tag{11}$$

To further allow for the innovations to be correlated over time we assume

$$\varepsilon_N = (e_T \otimes I_N)\mu_N + \nu_N \tag{12}$$

Henceforth: $\sigma_1^2 = \sigma_\nu^2 + T\sigma_\mu^2$ and $\theta = 1 - \sigma_\nu/\sigma_1$.
Although the two data generating processes look pretty similar, they imply different spatial spillover mechanisms governed by the different structure of the implied variance covariance matrix.

Estimation

The estimation procedure is a combination of the traditional panel data literature on error component models and the GM approach to spatial models. More in detail:

- [Kapoor et al., 2007] suggest three GM estimators of ρ and the variance component terms (controlled by the argument `method`):
  - The first set of GM estimators is based only on a subset of the moment conditions (should be considered only an initial estimator)
  - The second set of GM estimators uses all moment conditions and an optimal weighting matrix (the inverse of the variance-covariance matrix of the sample moments at the true parameters values)
  - The third set of GM estimators is motivated by computational difficulties: It uses all moment conditions but not an optimal weighting matrix
- Once we have estimates for the spatial autoregressive coefficient and the two variance components we can estimate the parameters vector by GLS
- The GLS estimator is identical to an OLS calculated on a "doubly" transformed model (spatial Cochrane-Orcutt and error component literature transormations)

RE model - GM Estimation

The function that performs the GM estimation of the model described above is `spregm`. The syntax is illustrated in the following example:

```
> GM <- spregm(fm, data = Produc, w = usaww, method = "fulweigh")
```

RE model - GM Estimation

The summary method produces the following output

```
> summary(GM)

Spatial panel random effects GM model

Call:
spregm(formula = fm, data = Produc, w = usaww, method = "fulweigh")

Residuals:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
-0.087800 -0.012900 -0.000327  0.000125  0.013100  0.097100

Estimated spatial coefficient, variance components and theta:
            Estimate
rho       0.55388299
sigma^2_v 0.00059157
sigma^2_1 0.03438935
theta     0.86884359

Coefficients:
              Estimate Std. Error t-value  Pr(>|t|)
(Intercept)  1.3389548  0.2122478  6.3085 2.818e-10 ***
log(pcap)    0.0607915  0.0525164  1.1576   0.24704
log(pc)      0.4391763  0.0442279  9.9298 < 2.2e-16 ***
log(emp)     0.5734594  0.0438597 13.0749 < 2.2e-16 ***
unemp       -0.0073402  0.0031544 -2.3269   0.01997 *
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1
```

SE model - GM Estimation

There is also a function that deals with the estimation of a system of spatially autocorrelated simultaneous equations [Kelejian and Prucha, 2004]. We do not discuss the theory but show how to use it.
Let us first create the data:

```
> W <- nb2listw(cell2nb(10, 10))
> y <- matrix(rnorm(200), 100, 2)
> X <- matrix(runif(200, -10, 10), 100, 2)
> ynseq <- paste("y", seq(1, ncol(y)))
> colnames(y) <- ynseq
> xnseq <- paste("y", seq(1, ncol(X)))
> colnames(X) <- xnseq
> data <- cbind(y, X)
> data <- data.frame(data)
```

SE model - GM Estimation

Then, let us give a look at the syntax of the function

```
> se <- spsegm(y ~ X, spec = "default", listw = W)
```

SE model - GM Estimation

and, finally, the `summary` of the results:

```
> summary(se)
Symultaneous Equation Model:

Call:
spsegm(formula = y ~ X, spec = "default", listw = W)

Equation 1

 -------------------------------------------------------
             Estimate Std.Error t value Pr(>|t|)
W y 1       -0.593002 16.787834 -0.0353   0.9718
W y 2        0.157398  4.064513  0.0387   0.9691
y 2          0.055647  0.748704  0.0743   0.9408
(Intercept) -0.010134  0.417237 -0.0243   0.9806
Xy 1        -0.348051 42.396155 -0.0082   0.9934
Xy 2         0.846668 36.191499  0.0234   0.9813

 -------------------------------------------------------

Equation 2

 -------------------------------------------------------
             Estimate Std.Error t value Pr(>|t|)
W y 1        0.747052 18.296741  0.0408   0.9674
W y 2        0.284048  4.098257  0.0693   0.9447
y 1          0.036178  0.954448  0.0379   0.9698
(Intercept) -0.027414  0.446342 -0.0614   0.9510
Xy 1        -3.662630 37.587545 -0.0974   0.9224
Xy 2         2.394889 23.321522  0.1027   0.9182
```

Outline

What's next?

A lot of work still remains to be done

- Discover bugs: please help us !!!
- GM performs relatively well, ML needs to be improved
- Add recent available methodologies
- Move towards the theory of spatial dynamic panel data models

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